Scaling Effects In Elliptical Patches With Marginal Effects

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EXTENDED ABSTRACT

The problem of determining the benefit or cost of irrigation mosaics compared to a contiguous area of irrigation required the development of a suitable method. Here we describe a method which was developed using scaling of a property or process using power law scaling based on the area of the patch and given by:

$$f = C + \alpha_a a^{\beta_a} \alpha_b a^{\beta_b} \tag{1}$$

where C is the property that does not change with the area, a is the minor axis, b the major axis of an ellipse (Figure 1), α_x and β_x are empirical coefficients associated with the marginal impact of size on the property and $x = a$ or b.

This leads for elliptical patches to a result for the marginal impact change due to patches compared to one contiguous area (I_{Re}) where the total areas are the same given by:

$$\frac{I(a,b)}{I(A,B)} = I_{\mathrm{Re}} = \frac{C + \gamma(a^{\beta_a} b^{\beta_b})}{C + \gamma(A^{\beta_a} B^{\beta_b})} = \frac{C + \gamma(ab)^{\beta}}{C + \gamma n^{\beta}(ab)^{\beta}} \tag{2}$$

where $I(a,b)$ is the patch impact and $I(A,B)$ is the contiguous area marginal impact, a and A are the minor axis of the patch and contiguous are ellipse respectively, b and B are the major axis of the patch and contiguous are ellipse respectively, $\gamma = \alpha_a \alpha_b$ and n is the number of small patches.

For a circle then $r = a = b$ and eqn (2) reduces to:

$$I_{\mathrm{Re}} = \frac{C + \gamma r^{2\beta}}{C + \gamma n^{\beta} r^{2\beta}} \tag{3}$$

These equations show that the marginal benefit or cost is dependent on β and this provides a great deal of utility in determining whether a benefit or cost will accrue for a particular process. When $\beta = 0$ then $I_{\mathrm{Re}} = 1$and there is neither cost nor benefit from irrigation mosaics. When $\beta < 0$ then there is a cost for irrigation mosaics and $\beta > 0$ means that a benefit arises from irrigation mosaics. This scaling approach should prove a useful tool decision making processes where systems such as irrigation mosaics are being considered.

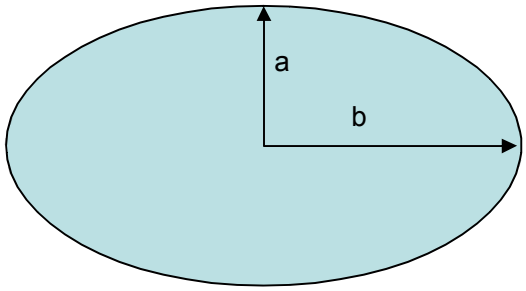

Figure 1. Ellipse with characteristic major axis, a and b. The marginal impacts are Δa and Δb.

Examples using; water table rise, groundwater mound spreading, groundwater solute spreading and the wind function for evaporation are presented and give values of β ranging from 0.57 to -0.05. This approach to landscape scaling may also be useful for some ecological processes.