Exploring the Applicability of Richards' Equation Models Using Multiple Observations

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EXTENDED ABSTRACT

Richards' equation models have been extensively applied to simulate water transport in soils. These models are usually calibrated against a set of observed field or laboratory time series. However, the models can yield dramatically different predictions for conditions outside the range of the calibration data, or the models can give inconsistent prediction on variables that are not used in the calibration. Insight into the appropriateness of Richards' equation models for a particular soil type can be obtained by model evaluation against multiple observations. Simultaneous soil moisture (time domain reflectometry) and deep percolation data from a lysimeter experiment on two soil types (a heavy and a light soil) in south-eastern Australia were used. Lysimeters represented 2.2 m deep undisturbed soil cores with a constant water table depth of 1.8 m. Perennial pasture was established in the cores and which was regularly irrigated. Monte Carlo simulations of the HYDRUS-1D model were applied for each soil type to simulate water transport in the lysimeters. Each Monte-Carlo simulation provided a model realization with a set of model parameters (the crop factor and eight soil hydraulic parameters) sampled from a feasible range of parameter values. Then model performance for each Monte Carlo realisation was evaluated against soil moisture and deep percolation data using the root mean square error as performance measure. The Richards' equation model was not able to provide consistent predictions of both soil moisture and deep percolation for the heavy soil. Only soil moisture was well predicted, deep percolation predictions had substantial errors for all model realizations. This means that calibrating the model parameters using soil moisture data is unlikely to result in useful predictions of deep percolation. The Richards' equation model performed better when used to simulate water transport in the light soil type. Both soil moisture and deep percolation were well predicted on this soil type. Therefore it is possible to obtain reliable model parameters for deep percolation predictions when calibrating against soil moisture data. The 1-D Richards' equation appears to be a physically sound model of water transport for this light soil.

1. INTRODUCTION

Models simulating water transport in soils based on the Richards' equation are widely applied in irrigation management and design (e.g. Thorburn et al. 2003).

The applicability of Richards' equation models is often limited by difficulties in field parameterization (both calibration and measurement of model parameters) and the often poor representation of preferential flow processes in soils (Walker and Zhang, 1999). Richards' equation models that explicitly include preferential flow, represent water flow in soils more realistically. However, parameterization for these models is even more difficult than for models that do not account for preferential flow. Although these limitations have been recognised for many years, there is still little systematic knowledge available about how appropriate Richards' equation models are for representing water transport on different soil types.

Insight into the appropriateness of Richards' equation models can be obtained by model evaluation against multiple observations (i.e. state variables and fluxes). Few field data sets are available that allow this type of analysis. Lysimeters provide an opportunity to observe water transport under quasi-field conditions (Bond, 1998).

In this study simultaneous soil moisture and deep percolation data from a lysimeter experiment in south-eastern Australia were used to evaluate the performance of a Richards' equation model for a heavy and light soil type. It is investigated whether reliable deep percolation estimates can be obtained from model calibration against soil moisture data.

2. MATERIAL AND METHODS

2.1. Lysimeter Data

The two soils investigated were part of a lysimeter experiment in Tatura (south-eastern Australia). The lysimeter experiment was conducted to quantify response of deep percolation to different soil types, water table depths in the Shepparton Irrigation Region and ponding times during surface irrigation. The lysimeter experiment is detailed in Bethune et al. (2007).

The two soils studied were a heavy Lemnos loam and light Sandmount sand soil type. The Lemnos loam was characterised by a 300 mm deep topsoil (silty clay loam) and a subsoil with low hydraulic conductivity (clay). The Sandmount had a 100 mm deep topsoil (sandy loam) and a clayey sand/ sandy clay subsoil.

Lysimeters represented undisturbed soil cores, each of 0.75 m diameter and 2.2 m depth. Perennial pasture was established in the cores, consisting of ryegrass and white clover.

Lysimeters had a water table depth of 1.8 m which was set by applying a constant water pressure to the base of the lysimeters using Mariotte bottles.

Irrigation events were triggered when accumulated US Class A pan evaporation minus rainfall, since the last irrigation, exceeded 50 mm. An irrigation event consisted of maintaining a pond of water on the lysimeter surface for a period of time (i.e. irrigation ponding time). After this time, remaining surface water was drained and measured as runoff. A total of 18 irrigation events were applied during the irrigation season 2005/2006 (from October 2005 until May 2006). Deep percolation was measured as the difference between water leaving (percolation) and entering (capillary rise) the base of the lysimeter, with net water leaving the lysimeter recorded as positive deep percolation. Total deep percolation for the irrigation season varied considerably between the two lysimeter cores. Sandmount sand and the Lemnos loam had 2770 mm and 9 mm of deep percolation, for irrigation season 2005/2006 respectively (Figure 1).

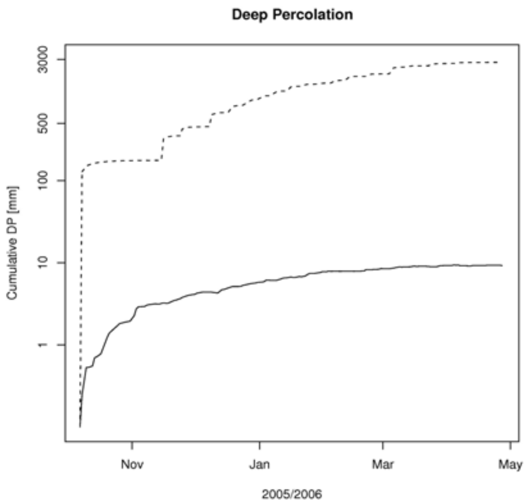

Figure 1. Cumulative deep percolation for the Lemnos loam (solid line) and the Sandmount sand (dashed line).

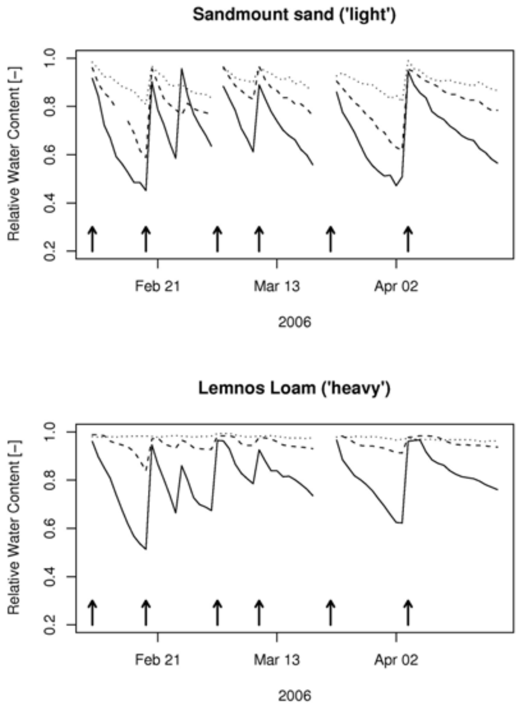

Figure 2. Time series of relative water contents (i.e. water contents normalised by maximum water content measured in a particular depth) measured by TDR for selected soil depths: 0.1 m (solid line), 0.3 m (dashed line) and 0.6 m (dotted line). Arrows indicate selected irrigation events during season 2005/2006.

Rainfall was measured at a Bureau of Meteorology Climate Station, located within 100 m of the lysimeter facility. Additional climatic data were used to calculate the daily crop evapotranspiration (Allen et al., 1998). The soil moisture profile for each lysimeter was measured at depths of 0.1, 0.2, 0.3, 0.4, and 0.6 m using time domain reflectometry (TDR). Soil moisture responded to irrigation and rainfall at all depths in the Sandmount sand (Figure 2). Lemnos loam had a similar response to irrigation and rainfall, however redistribution was slower than for the Sandmount sand. Soil moisture below 0.3 m soil depth was less responsive than for Sandmount sand.

2.2. Model Simulations

The one-dimensional Richards' equation model HYDRUS-1D (Simunek et al., 2004) was applied to simulate the water flow in the lysimeters. A 2.2 m deep soil profile was specified, consisting of two soil horizons (i.e. topsoil and subsoil). Model boundary and initial conditions and model parameters were specified as follows.

The atmospheric boundary condition was specified as a daily flux (precipitation plus irrigation minus runoff). Precipitation, irrigation and runoff were measured during the experiment. Water was not allowed to build up on the soil surface and any excess water from rainfall or irrigation that did not infiltrate was removed instantaneously. However, to be consistent with the lysimeter experiment, model realizations with runoff greater 1 % of potential infiltration (i.e. irrigation plus rainfall minus runoff) were not considered for further analysis. Soil evaporation was neglected (full plant cover). Simulations with relative mass balance error (i.e. absolute error divided by sum of all boundary fluxes) of greater than 1% and 0.05 % for Sandmount sand and Lemnos loam, respectively were discarded. This difference accounted for the different amount of deep percolation on these two soil types. Computation of root water uptake required estimates of the potential evapotrans-piration. The potential evapotranspiration was calculated as the product of reference evapotranspiration (Allen et al., 1998) and a crop factor. The root water uptake was computed from the potential evapotranspiration, a typical root distribution of mixed clover and grass pasture reported by Mehanni and Repsys (1986) (i.e. 65%, 22%, 8%, 3% and 2% of the roots located in 0-0.1, 0.1-0.2 , 0.2-0.3, 0.3-0.45 and 0.45-06 m soil depth, respectively) and soil water stress function for surface irrigated pasture (*P0* = 0 cm, *POpt* = 0 cm, *P2H* = -200 cm, *P2L* = -800, *P3* = -8000 cm, *r2H* = 0.5 cm/d, *r2L* = 0.1 cm/d) (Bethune and Wang, 2004), and the distribution of the pressure head calculated by the model. A fixed pressure at the bottom of the lysimeter core of 40 cm was considered as the lower boundary condition to represent the lysimeter's water table depth of 1.8 m. Measured water contents at soil depths of 0.1, 0.2, 0.3, 0.4 and 0.6 m were used as initial conditions.

Soil hydraulic properties (water retention and hydraulic conductivity curves) were represented using the models and parameterization of Mualem (1976) and van Genuchten (1980).

Ten-thousand Monte Carlo simulations of HYDRUS-1D were applied for each soil type using Latin hypercube sampling from a uniform parameter distribution. Parameters sampled were: the crop factor and eight soil hydraulic parameters, i.e. *alpha* (a scaling parameter), *n* (shape of the curve), tortuosity parameter *l* of the conductivity function and the saturated hydraulic conductivity for the topsoil and the subsoil. The parameters were uniformly sampled with allowable ranges: *alpha* (0.001…0.2 1/cm), *n* (1.05…1.7), tortuosity parameter *l* (-3…3), saturated conductivity of topsoil (9…1000 cm/d) and saturated conductivity of subsoil (0.02…220 cm/d). A logarithmic parameter distribution was used for sampling of the saturated conductivity. Saturated water contents were fixed to average maximum water contents measured at all depths within a particular soil horizon. Residual water contents were assumed to be zero.

Each Monte Carlo simulation was evaluated using daily soil moisture data measured at five depths (0.1, 0.2, 0.3, 0.4 and 0.6 m) and deep percolation measured for 18 irrigation events during the irrigation season 2005/2006. The performance of each model realization was assessed using root mean square error RMSE. Model discretization and iteration criteria as recommended by J. Simunek (http://www.pc-progress.cz/_forum/) were used.

3. RESULTS AND DISCUSSION

Monte Carlo simulations revealed that, for both soil types, most model parameters were poorly constrained by soil moisture data, i.e. a wide range of parameter combinations predicted the soil moisture data similarly well. Studies of Jacobsen and Schjonning (1993) and Lane and Mackenzie (2001) suggest that the standard error of TDR measurements in the field can be as great as 0.03 m^3/m^3 when appropriate calibration is used. These errors can arise from difficulties in the interpretation of the TDR trace and small scale soil heterogeneity of texture, bulk density and temperature. There may also be other errors such as an imperfect model structure. Therefore, all model realizations with RMSE < 0.05 m^3/m^3 were interpreted as reasonable model calibrations, i.e. they predicted soil moisture data similarly well. Only three to four out of 9 parameters: topsoil *alpha*, subsoil *n* and crop factor (for Lemnos loam) and saturated conductivity, *alpha* and *n* of subsoil, topsoil *n* (for Sandmount sand) were slightly constrained by soil moisture data (Figure 3, Lemnos loam not shown).

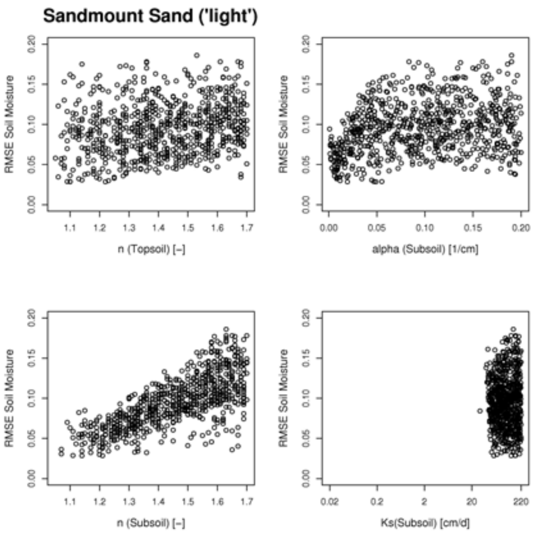

Figure 3. Scatterplot relationship between model parameters and RMSE of the soil moisture predictions from 10,000 Monte Carlo simulations (Latin Hypercube sampling) using HYDRUS-1D. Each dot represents a single model realization and its corresponding RMSE.

For the heavy Lemnos loam soil type, the Richards' equation model provided a reasonable prediction of soil moisture measurements, with one model realizations of RMSE < 0.05 m^3/m^3. However, the model could not provide realistic simultaneous predictions of both soil moisture and deep percolation (Figure 4). The best model realization had deep percolation predictions of RMSE = 17 mm per irrigation event whereas deep percolation sums during the entire irrigation season (18 events) were only 9 mm (Figure 1). The model realization that predicted soil moisture data best had RMSE = 37 mm for deep drainage. This indicates that calibration using soil moisture data (often available in the field) is unlikely to provide reliable predictions of deep percolation on a Lemnos loam soil type. The Richards' equation approach may be inappropriate for these cracking soils, where infiltration is dominated by preferential flow through soil cracks (Prendergast 1995).

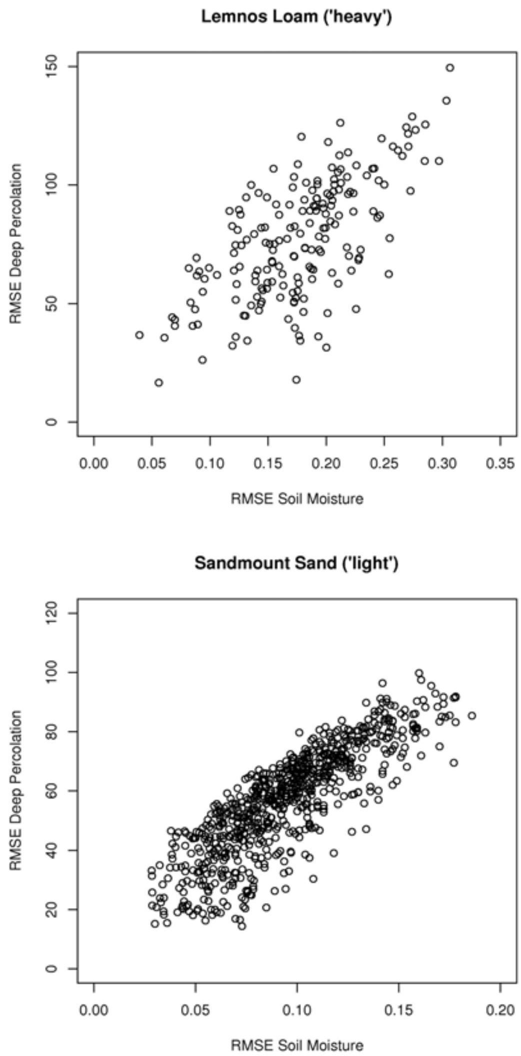

Figure 4. Scatterplot relationship between RMSE of soil moisture and deep drainage predictions from 10,000 Monte Carlo realisations with Latin hypercube sampling using HYDRUS-1D. Each dot represents a single model realization and its corresponding RMSE.

The light Sandmount sand soil type had 49 model realizations with RMSE < 0.05 m^3/m^3 for soil moisture data (Figure 4). These simulations had prediction errors for deep percolation ranging from 15 mm to 47 mm. Average deep percolation was 154 mm per irrigation event for this soil type. Therefore it is possible to obtain reliable predictions for deep percolation when the model parameters were calibrated against soil moisture data. The 1-D Richards' equation appears to be a physically sound model of water transport for this light soil.

4. CONCLUSION

Soil moisture and deep percolation data measured in a lysimeter experiment were used to test the appropriateness of the Richards' equation models for representing water transport on a heavy and a light soil type.

The 1-D Richards' equation model was appropriate for the light Sandmount sand soil type. Calibration using soil moisture is likely to give reliable model parameters.

The Richards' equation model used in this study did not provide a realistic representation of water transport on the heavy Lemnos loam soil type. One cannot obtain reliable predictions of deep percolation by calibrating model parameters against soil moisture data.

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