Designing an Engineering Style Hydrological Model to Advance a CGE Model of a Multi-Regional Economy

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EXTENDED ABSTRACT

Storages of water available to many economies have been decreasing for a number of years. With the possibility of climate change reducing inflows as well as increasing populations increasing demands, inventory amounts are expected to be further strained in years to come. As a consequence efficient management of water supplied for consumption and maintenance of inventory amounts for future consumption has become increasingly more important. Furthermore water is an essential commodity in all economies around the world. Thus, with increasing variance in supplies effects on economies are becoming more pronounced. This paper is concerned with ideas for a water supply network model. In particular the REALM (Diment, 1991) style model is explored.

The current model used by for Victorian water authorities is REALM (REsource ALlocation Model). The model works on a water supply network. This network consists of a number of nodes for example:

- Demand Centres (both Urban and Rural) which are points of consumption
- Reservoirs which supply and store water

These nodes are connected by water carriers of two types, pipe and/or rivers. Water moves to and from these nodes via water carriers.

The REALM style model achieves desired objectives by minimising a cost or penalty function in a given time-step. Inputs required for the model for future time-steps are expected:

- Climate data
- Water demanded (may be restricted when necessary)
- Starting inventory amounts (initial reservoir volumes)

The model performs a cost minimisation at each time-step and provides expected outputs for future time-steps:

- Water supplied
- Inventory amounts (reservoir volumes)

The cost or penalty function in REALM is applied to volumes of water. Depending on the given penalty type the cost function will increase linearly with that volume of water. It is because of this linear cost/volume relationship that REALM is able to use a fast network linear programming algorithm RELAX (Bertsekas, 1991) to find particular flows at minimum cost in each time-step. In terms of models REALM has the capability of determining rather fast solutions for flows in large water supply networks. The introduced hydrological model for water supply network will be based on current models, in particular REALM. One aim is to improve the quality of solutions by incorporating more descriptive cost functions. These necessarily may be non-linear with regard to water volumes. Solutions are obtainable applying numerical methods of optimisation which are now computationally viable to solve. Another aim is for a flexibility in the cost functions enabling a wide variety of solutions. This paper explores the concepts behind these cost functions.

The model born will be integrated with a computable general equilibrium model. The aim for a general equilibrium model is to analyze the impacts of economic changes. With water an essential commodity significant changes in supplies will result in significant economic consequences.

1 INTRODUCTION

This paper is concerned with ideas for a water supply network model. In particular the REALM (Diment, 1991) style model is explored. The REALM style model achieves desired objectives by minimising a cost or penalty function in a given time-step. A cost or penalty is incurred if a given objective is not met. Cost or penalty functions in REALM are applied to volumes of water. Depending on the given penalty type the cost function will increase (decrease) linearly with that volume of water incurring the penalty. This paper introduces conceptual ideas for a water supply network model. One aim is to improve the quality of solutions in a model by incorporating more descriptive cost or penalty functions. These necessarily may be non-linear with regard to water volumes. Another aim is for a flexibility in the cost functions enabling a wide variety of solutions. The model born will be integrated with a computable general equilibrium model.

REALM is a generalised simulation computer software package that models harvesting and bulk distribution of water resources within a water supply system (Perera et al., 2003). REALM is currently used in determining water allocations in Victoria but can be implemented on any water supply network. It is an engineering style model developed by Diment (1991) and furthered by Perera et al. (2000, 2003, 2005). REALM uses a fast network linear programming algorithm for optimisation of water delivery (allocation) to different demand nodes.

Schematically, the major components of REALM are input processing, simulation and output processing (Perera at al., 2005). Required inputs are in the form of streamflow, demand and system files. These are respectively the expected future climate data, demands and network structure. These variables are considered as external inputs and could be modelled independently. Simulation involves:

- definition of run time parameters
- getting information from set up
- simulation step, which involves satisfying demands, minimising spill, meeting reservoir targets and satisfying in-stream requirements.

Main outputs from the model are expected future supplies and inventory amounts. Output files are in the form of a time series of different system characteristics, including carrier flows, reservoir levels, restricted demands etc. REALM can operate on a daily, weekly or monthly basis. Water demand in the REALM system is modelled externally using the crop model called the Program for Regional Irrigation Demand Estimation (PRIDE), which will not be considered in the present paper and the demands are treated as totally exogenous variables of the model. This pre-processing program uses crop area and crop type data at irrigation nodes as basic input. The maximum irrigable areas for different crop types remain fixed during a model run and supplies are restricted in response to reduced allocations during droughts. Various scenarios of structural change to irrigated land use can be modelled through modifications to the crop area and crop type inputs for different model runs (Weinmann et al., 2005).

REALM incorporates the RELAX software (Bertsekas, 1991) which uses an objective function that minimises the sum of flow times penalty in the modelled network to obtain an optimised distribution of flows each time step while not exceeding any capacity constraints and also achieving a water balance at each node. A set of convergence criteria and tolerances is used to determine when the solution has converged to an acceptable accuracy. Convergence criteria is required due the concept of target storage and total system storage values at the end of the time-step. That is these values are assumed in the optimisation and updated until all variables converge.

The aim of this paper is to discuss a water supply network model that chooses flows resulting in a cost minimisation and water balance in a given time-step. In particular the concepts behind these cost functions and water balance conditions are explored. The model will be aligned with REALM, discussing how the engineering style model functions. This alignment is not in the usual presentations of REALM. REALM uses many different penalty functions however when combined they simplify to just three types. These ideas form the basis for a water supply network model whose output of water supplied and inventory amounts can then be used for a general equilibrium model.

2 SIMPLE WATER SUPPLY NETWORK

This simplified model structure can be used to determine water flows across a water supply network by a cost minimisation during a given time-step. It requires a given set of initial conditions (reservoir volumes) and expected climate data and demands. Ultimately the model can be run over an arbitrary amount of time-steps provided climate data and demands are available updating reservoir volumes at the end of each time-step.

In describing the network *carriers* are defined as water movers eg. rivers and pipes. Carriers have maximum and minimum capacities and additionally may experience water losses expressed as percentage of actual flow. *Nodes* are defined as locations that experience water inflows or outflows or both along carriers. At this stage just two types of nodes are described, reservoirs and demand centres. Reservoirs might experience natural inflows (rainfall) or outflows (evaporation) as well as both carrier inflow and outflow and spilling. Demand centres only experience carrier inflow. Reservoirs have maximum and minimum capacities, demand centres have a given demand for water.

The model has penalty or cost function over the network which will ultimately be a function of the carrier flows. Carrier flows during each time-step are selected so as to minimise cost. The cost function includes costs of regular flows along carriers, cost of failure to meet demands and cost of reservoir volumes deviating from acceptable levels.

Long term maintenance of the water supply (inventory amounts) is usually achieved by restricting demands. That if is the current system storage is significantly low then the demand of the given demand centers is reduced to that which is deemed acceptable. In the following model assume that need for restrictions has been checked, that is the demands have been restricted if necessary.

Defining the physical elements and sets of the model, let

$$\begin{aligned} \mathcal{R} &= \text{the set of reservoirs} \\ i \in \mathcal{R} &\Leftrightarrow i \text{ is a reservoir} \\ \mathcal{D} &= \text{the set of demand centres} \\ i \in \mathcal{D} &\Leftrightarrow i \text{ is a demand centre} \\ \mathcal{C} &= \text{the set of carriers} \\ (i,j) \in \mathcal{C} &\Leftrightarrow (i,j) \text{ is a carrier from node } i \text{ to } j \end{aligned}$$

Defining the variables of the model, let,

$$\begin{aligned} x(i,j,t) &= \text{the flow along carrier from node } i \text{ to node } j \text{ during time-step } t \\ d(i,t) &= \text{the volume of shortfall in demand for demand centre node } i \text{ in time-step } t \\ v(i,t) &= \text{the volume in reservoir node } i \text{ in time-step } t \end{aligned} \tag{1}$$

Finally defining the cost or penalty functions, let

$$\begin{aligned} c_{ij}(x(i,j,t)) &= \text{penalty for flow } x(i,j,t) \text{ along carrier } (i,j) \\ s_i(d(i,t)) &= \text{penalty for demand shortfall } d(i,t) \text{ in demand centre node } i \\ r_i(v(i,t)) &= \text{penalty for reservoir volume } v(i,t) \text{ in reservoir node } i \end{aligned} \tag{2}$$

So values for the variables $x(i,j,t), d(i,t), v(i,t)$ must be chosen so as to minimise the following,

$$\sum_{(i,j)\in\mathcal{C}} c_{ij}(x(i,j,t)) + \sum_{i\in\mathcal{D}} s_i(d(i,t)) + \sum_{i\in\mathcal{R}} r_i(v(i,t)) \tag{3}$$

Subject to domain constraints on the variables,

$$\forall\ (i,j) \in \mathcal{C},\quad 0 \le x(i,j,t) \le C(i,j,t) \tag{4}$$
$$\forall\ i \in \mathcal{D},\quad 0 \le d(i,t) \le D(i,t) \tag{5}$$
$$\forall\ i \in \mathcal{R},\quad 0 \le v(i,t) \tag{6}$$

Where exogenous to the time-step parameters,

$$\begin{aligned} C(i,j,t) &= \text{maximum capacity of carrier } (i,j) \text{ in time-step } t \\ D(i,t) &= \text{demand for demand centre node } i \text{ in time-step } t \end{aligned} \tag{7}$$

The water must be balanced at each of the nodes which leads to the following additional constraints.
$\forall\ i \in \mathcal{R}$,

$$\begin{aligned} v(i,t) = W(i,t) - E(i,t) + I(i,t) \\ - \sum_{j|(i,j)\in\mathcal{C}} x(i,j,t) \\ + \sum_{j|(j,i)\in\mathcal{C}} (1 - \lambda(j,i,t))\, x(j,i,t) \end{aligned} \tag{8}$$

where exogenous to the time-step parameters,

$$\begin{aligned} W(i,t) &= \text{start reservoir volume for reservoir node } i \text{ in time-step } t \\ I(i,t) &= \text{inflows to reservoir node } i \text{ in time-step } t \\ E(i,t) &= \text{outflows (evaporation) to reservoir node } i \text{ in time-step } t \\ \lambda(i,j,t) &= \text{percentage of flow lost in transmission from carrier } (i,j) \text{ in time-step } t \end{aligned} \tag{9}$$

In words (8) states for each reservoir node in the time-step:

End Volume (Includes Any Spills) =
Start Volume − Outflows + Inflows
− Carrier Outflows + Carrier Inflows

$\forall\, i \in \mathcal{D}$,

$$\sum_{j\in\mathcal{R}}(1-\lambda(j,i,t))\,x(j,i,t)+d(i,t)=D(i,t) \quad (10)$$

In words (10) states for each demand centre node in the time-step:

Carrier Inflows + Shortfall = Demand

The minimisation of (3) yields flow solutions and thus supplies for the network in the time-step.

Remark: It may have become apparent that the reservoir volume $v(i,t)$ did not have a maximum bound. The reason for this is to account for spills (water lost to the system). Define $W_{max}(i,t)$ as the maximum volume of reservoir node i in time-step t. Then,

$$\begin{aligned} v(i,t) < W_{max}(i) \quad &\Rightarrow W(i,t+1)=v(i,t)\\ v(i,t) \ge W_{max}(i) \quad &\Rightarrow W(i,t+1)=W_{max}(i) \end{aligned} \quad (11)$$

After the adjustment of reservoir volumes in (11) inventory amounts are available.

$$\begin{aligned} L(t) \;=\; &\sum_{i\in\mathcal{R}}(v(i,t)-W_{max}(i))\\ &\times[I(v(i,t)-W_{max}(i)>0)] \end{aligned} \quad (12)$$

where $L(t)$ is defined to be spills water lost to the system in time-step t and $I(A)$ refers to the indicator of the set A, i.e. $I(A)=1$ if event A has occurred, and equals to 0 otherwise.

3 PENALTY FUNCTIONS

In model thus far three penalty functions defined in (2) have been introduced. These functions can be chosen to suit the model and the objectives it would like to achieve. In this section conceptual ideas for the penalty functions are introduced.

3.1 Reservoirs

In order manage and preserve stores of water for future consumption it is important that reservoir volumes are kept at acceptable levels. That is given the total system storage at the end of the time-step, $S_T(t)$, the user might have a preferred distribution of this water across all reservoirs. That is, define $S(i)$ as preferred volume in reservoir i, then

$$S_T(t)=\sum_{i\in\mathcal{R}}S(i) \qquad 0\le S(i)\le W_{max}(i) \quad (13)$$

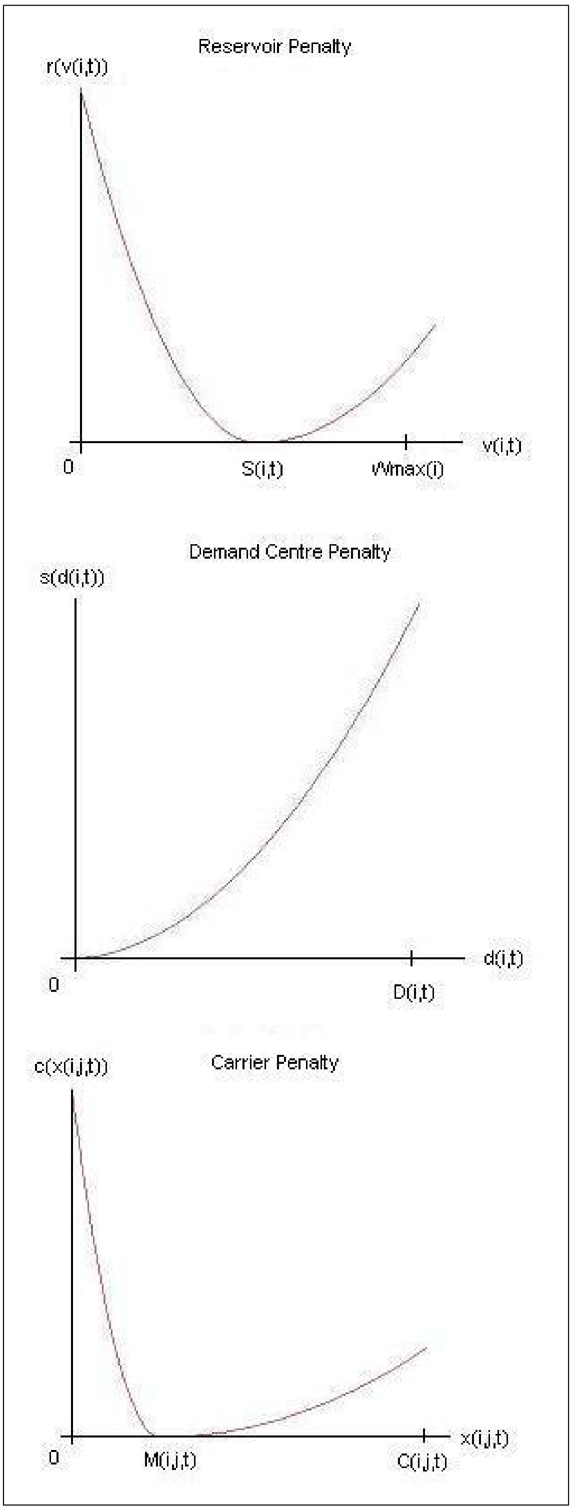

Figure 1. General Model Reservoir, Demand Centre and Carrier Penalty Examples

For all reservoirs i the model should penalises any movement away from this preferred volume $S(i)$. For example the generalised penalty function illustrated in Figure 1 would do this. Furthermore given a reservoir will spill for any volume above $W_{max}(i)$ the model must also penalise these spills as in Figure 1. Notable also iswhen the reservoir is empty the highest penalty is incurred.

3.2 Demand Centres

In order to maintain reliability in supply for users the model must penalise any shortfall in meeting the demands. Figure 1 illustrates a penalty function for demand shortfalls. Note the highest penalty incurred when none of the demand is met, no penalty when demand is met.

3.3 Carriers

Carriers, particularly rivers, may have minimum flow requirements. River minimum flow requirements are referred to as environmental flows. Any failure to meet these minimum flows should be penalised. Furthermore there may be costs attributed to flows exceeding the minimum flows. Figure 1 illustrates a penalty function for carrier flows where $M(i, j, t)$ is the minimum flow requirement in carrier (i, j) in timestep t.

4 ALIGNING WITH REALM

As mentioned earlier REALM is the model currently used in determining water allocations in Victoria. In the usual presentation of REALM it is viewed as a linear network program. However it is presented slightly differently here.

The penalty functions of the REALM style model can be represented as piecewise linear functions as presented in Figure 2. REALM is flexible as to how many 'pieces' there are.

REALM adopts a fast linear network programming algorithm RELAX. In order to implement this algorithm REALM converts the water supply system to a an equivalent component network (Perera et al., 2003). Effectively the presented model of carriers, nodes and three penalties is represented as a system of arcs and nodes with numerous penalties. For every line segment in the penalty functions in Figure 2 an arc is formed by REALM. Most of these arcs are non-physical. That is they are not a physical carrier in the sense of rivers and pipes. They serve more of an accounting purpose by converting the volumes in the dependent variables of the penalty function to arc flows with penalties attributed to those arc flows. The magnitude of the gradient a line segment in a penalty function is the size of the penalty per unit of flow along the arc. The capacity of the arc is the length of the segment of the dependent variable in one line segment.

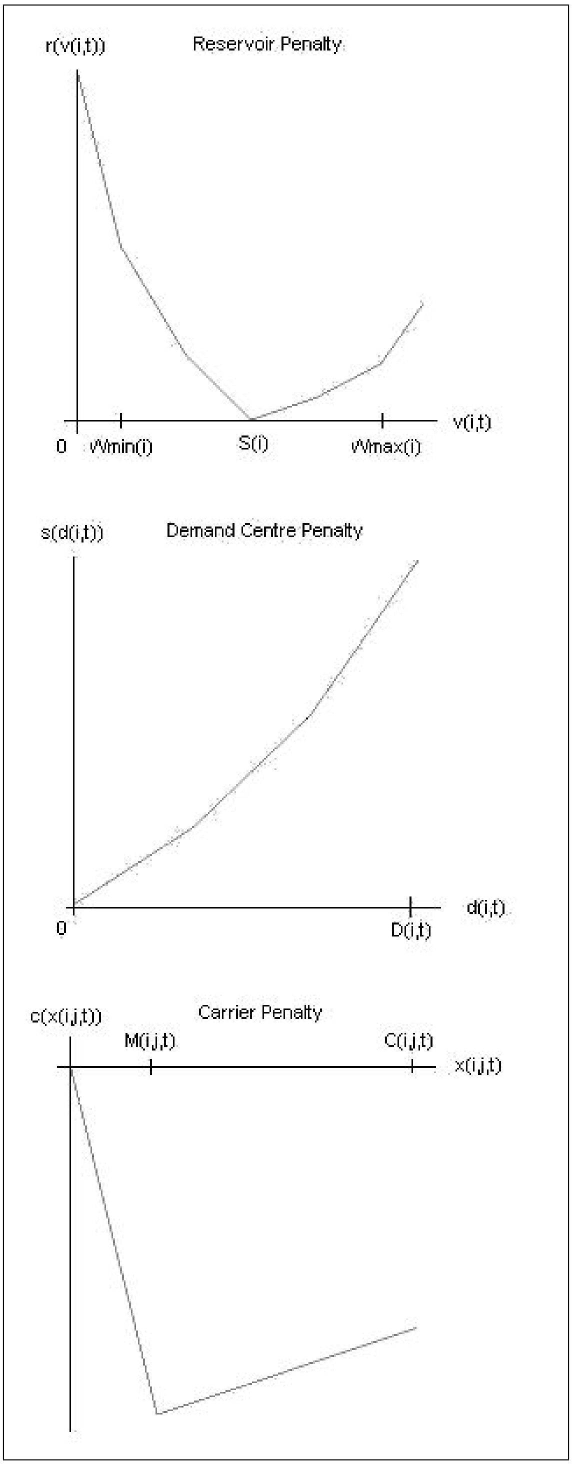

Figure 2. REALM Style Reservoir, Demand Centre and Carrier Penalty Examples

It may also become apparent from Figure 2 that if a minimum flow requirement exists for a carrier then the penalty function for the REALM style model is down-shifted. This makes no difference as to where the global minimum occurs, just to the value of the global minimum. REALM does this in order to implement this penalty within the arc structure. The first line segment in this penalty is represented as an arc with a negative penalty equal to that of the gradient of the line segment. This serves to encourage the minimum flow requirement to be met by ultimately reducing the overall penalty. As mentioned this arc/nodal structure then enables implementation of the RELAX algorithm.

At this point readers are referred to the referenced literature on REALM. It is hoped that readers can relate the current presentations available of REALM to the penalty functions presented in Figure 2.

5 DISCUSSIONS AND CONCLUSIONS

The configuration of the hydrological model presented is a general structure which can be built on. In particular there is flexibility in choosing the penalty functions which ultimately determine the solutions. A relationship between the general model and the REALM model has been shown. Unfortunately all but carrier penalties are hard coded into REALM, although there is room to marginally alter the linear penalty sizes to achieve specific optimisations.

Within the model it is an aim to have penalty functions (2) as non-linear functions of input variables. REALM can be used to calibrate the model whilst still leaving a flexibility to deviate from REALM obtainable solutions. It is envisioned that solving the model will become computationally more expensive. Just how expensive will depend on the complexity of the penalty functions and the number of nodes and carriers present in the water model. However with advances in computation time and new numerical methods it is also envisioned that the problems of 1991 (REALM's development) may not be as relevant today.

The hydrological model will then be used in a computable general equilibrium model (CGE) for a multi-regional economy as described by Dixon et al. (2005). Inclusion of the hydrological model described in the present paper to the CGE model will allow researchers to implement convincing policy analysis of a wide range of important issues including the following.

Competing urban and rural water demands: While urban water usage is relatively small compared with irrigation water usage, competing demands will increasingly become an issue due to projected relatively rapid population growth in coastal urban areas of Australia. One probable water source for some urban regions will be water purchased from other regions. In addition to dealing with hydrological constraints on water trading, the model will provide a suitable framework for considering alternative investment decisions. For example, should investment be in pipeline construction to move water between regions, or should it be in urban-based desalination plants?

Climate change impact assessment: By making water supply a function in part of regional rainfall, we will be able to consider the potential impact that climate change will have on water supply at the regional level. This is a major issue in Perth, where a decline in average rainfall over the past few decades has resulted in a much larger proportional decline in water supplies.

Forestry development and associated increase in environmental flow demand: Forestry development increases the environmental demand for water, with potentially adverse impacts on regional economies. Greenhouse gas policies that encourage forestry may conflict with water policies. Analysis of such conflicts will be assisted by the model.

Assessment of economic impacts of farm dam development: Development of farm dams allows farmers to harvest the runoff on their property but reduces the total system discharge. The impacts on streamflow have been examined in hydrological studies (e.g., Schreider et al., 2002). The CGE approach will extend the analysis to examine the impact on regional economies. Dams may enhance the productivity of individual farms, but reduce water availability downstream.

Increase of irrigation efficiency: A paradoxical consequence of increased irrigation efficiency is that the total amount of water consumed in the system may increase. Poor irrigation practices result in a small proportion of irrigated water accumulating in crops or transpiring. Much of the rest returns to the river. Water allocation volumes at present are based on gross usage, not usage net of returns. Consequently, increased irrigation efficiency may result in net water usage increasing. In the CGE water modelling, net rather than gross water usage will be estimated.

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