

# Asset Market Equilibrium: A Simulation

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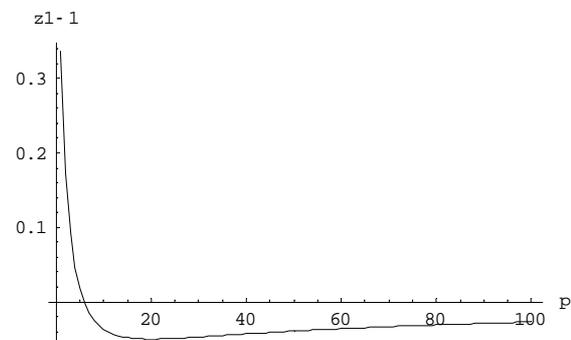
## EXTENDED ABSTRACT

This paper examines how the asset price is determined in the asset market, and how it changes through the modification of market structure, utilizing Lucas (1978) model, which incorporates *asset prices*. The main motivation in Lucas (1978) was the *existence* of equilibrium asset price, while the one in this paper is the *comparative statics*: the variation of asset price, especially when the dividend from the asset becomes riskier. The motivation of this paper is to examine if the standard asset price model can explain the *bubble economy* with risk averters. Historically, when the bubble emerged in an asset market, there existed the modification of preceding circumstances in many cases: *e.g.* in the Dutch tulip bubble. Thus, it examines if the asset price rises in the case of increased uncertainty when the investors are risk-averters, utilizing simulation approach. When the expected value of dividend rises the asset price may rise. However, when the expected value remains the same and the standard deviation (variance) increases, it might be expected that the investors' demand for asset decline, leading to the decline of asset price. When the investors plan to maximize the utility level under the *fixed* income, this expectation might be supported. In fact, in the general equilibrium approach of searching for equilibrium asset prices, the investor's income increases when the asset price rises: the well-known argument of *reservation demand*, such as in the textbook explanation of the labor supply and saving. Due to the *income effect* the expectation might prove to be wrong.

In Fukiharu (1994), which follows Lucas' formulation of the pure exchange general equilibrium model, it was shown that if the *relative* risk aversion of the investor's utility function does not exceed 1 and it is not decreasing, the equilibrium asset price declines when the uncertainty increases in the above sense. The assumption of *power function* satisfies the one in this result.

In this paper, following Lucas' formulation, we examine if there is any utility function, which

provides the counter example in which the asset price rises in spite of the increased uncertainty regarding the prospect for dividend receipt. Starting from the risk-averters two period maximization problem with *certain* dividends for the two periods, it is shown that if the *uncertainty* is introduced for the *first* period, the *exponential* utility function provides the counter example mentioned above. It is shown, however, that when the uncertainty is introduced for the *two* periods, the exponential utility function does not provide the case. Thus, when the uncertainty is not so strong, the income effect may raise the asset price in spite of the increased uncertainty. It is shown in this case that the asset is a *Giffen good* when the asset price is *already high*, as shown in the following figure, where  $p$  is the asset price and  $z_1-1$  is the excess demand for the asset.



Thus, this paper points out a theoretical possibility that even if the investors are risk-averters the bubble economy may emerge. In examining this problem, it is also found that *quadratic* utility function may explain the collapse of bubble economy

## 1. INTRODUCTION

The aim of this paper is to examine the asset price variation when the asset market structure changes. Historically, when the bubble emerged in an asset market, there existed the modification of preceding circumstances in many cases: *e.g.* in the Dutch tulip bubble. However, in the bubble economy, the investors appear to be risk-lovers. This paper focuses its attention on the explanation of bubble economy when the standard assumption is made: *i.e.* investors are risk-aversers. It is a standard exercise that assuming the existence of one risky asset and one riskless asset, when the investor's income increases, he or she reduces the *ratio* of investment on risky asset, raising the one on the risk less asset [Layard and Walters (1978, Chapter 13)]. This exercise is solved under the assumption of a quadratic utility function, a special type of risk-avertter assumption. Thus, it appears that under the assumption of the society with risk averters, the bubble of asset price cannot be explained in terms of this exercise. The exercise, however, does not imply the shrinkage of *absolute* demand for the risky asset. There might be the increase of *absolute* demand for the risky asset. Unfortunately, this exercise does not explain how the asset price changes.

This paper examines how the asset price is determined in the asset market, and how it changes through the modification of market structure, utilizing Lucas (1978) model, which incorporates the *asset price* determination. The main motivation in Lucas (1978) was the *existence of equilibrium* asset price, while the one in this paper is the *comparative statics*: if the asset price rises when the dividend from the asset becomes riskier, with the same expected value. Fukiharu (1994) obtained the conclusion in which with additional assumptions on the investors' relative risk aversion, the asset price falls. If these assumptions are not satisfied, what would happen? This paper examines if the asset price rises in this case, utilizing simulation approach.

## 2. CERTAIN ASSET DIVIDEND CASE

Lucas (1978) constructed a household's asset purchasing plan over infinite periods under the pure exchange model, examining the existence of equilibrium in the asset market. Since the purpose of the present paper is to examine the asset price variation under the modified circumstance, we simplify the Lucas model as much as possible. In this section, we assume that

there is the aggregate household, planning the optimum asset possession and goods consumption over two periods, under pure exchange model. There are one commodity and one asset. There is no production, so that the model may be well understood if we suppose that the asset is the foreign asset, and the household's consumption from the dividend is procured in the foreign trade. In this section one unit of the asset is assumed to yield the certain dividend,  $\underline{y}$ . The assumption of certainty of dividend allows the investor to consume the same  $\underline{y}$  for the two periods. In the first period, the aggregate investor possesses the asset on the amount of  $z_0=1$ . When  $p$  is the asset price, in terms of consumption good, the aggregate investor possesses the income on the amount of  $(\underline{y} + p) z_0$  in the first period. The aggregate investor plans the optimum consumption over the two periods:  $c_1$  and  $c_2$ , and the purchase of the asset,  $z_1$ , believing that the purchase of the asset on the amount of  $z_1$  guarantees the consumption on the second period,  $c_2 = \underline{y} z_1$ , while the asset is bequeathed in the second period. Assuming  $u(c)$  to be the utility function, the aggregate investor's behavior is expressed by the following maximization:

$$\begin{aligned} \max & u(c_1) + \beta u(c_2) \\ \text{s.t.} & c_1 + p z_1 \leq (\underline{y} + p) z_0, c_2 \leq \underline{y} z_1 \end{aligned} \quad (1-1)$$

where  $\beta$  is the discount factor.

From this maximization, we have demand functions,  $c_1(p)$ ,  $c_2(p)$ , and  $z_1(p)$ . The *certain pure exchange* stock market equilibrium is defined by  $p^*$ , which satisfies

$$z_1(p^*) = 1, c_1(p^*) = \underline{y}, \text{ and } c_2(p^*) = \underline{y} \quad (2-1)$$

We have the following result [Fukiharu (1994)]. This result is nothing but the fundamental theorem on *finance*; the asset price is the present value of the stream of dividends.

**Proposition 1:** Suppose that  $u'(c) > 0$  and  $u''(c) < 0$  ( $c > 0$ ). (3-1)

Then, we have the following.

- (i)  $z_1(p) < 1$  holds when  $\beta \underline{y} < p$ .
- (ii)  $z_1(p) > 1$  holds when  $\beta \underline{y} > p$ .
- (iii) Necessary and sufficient conditions for (2-1) to hold is  $p^* = \beta \underline{y}$ .

From *Proposition 1* it follows that  $p^* = \beta \underline{y}$  is the *stable* pure exchange stock market equilibrium. Assumption (3-1) is a standard one in microeconomics. In what follows, simulations are conducted by specifying utility function.

### 2.1 Power Function (I)—Certainty Case

One of the typical examples which satisfies (3-1) is the power function:

$$u(c)=c^\gamma \quad 0<\gamma<1. \quad (4)$$

The excess demand function for the asset when  $\gamma=1/2$ ,  $\beta=9/10$ , and  $\underline{y}=1$ :  $z_1-z_0$ , exhibited in Figure1, ascertains *Proposition 1*.

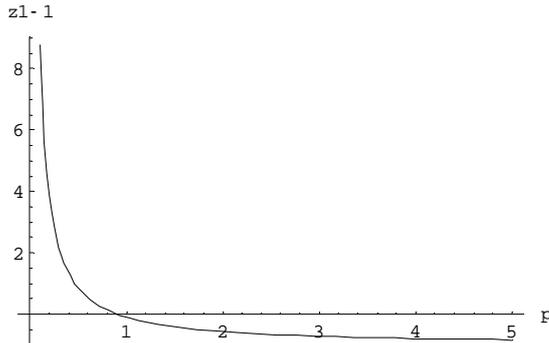


Figure1. Excess Demand Function (Power Function Case I)

### 2.2 Exponential Function I—Certainty Case

One of the other typical examples which satisfies (3-1) is the exponential function:

$$u(c)=1-e^{-\mu c} \quad \mu>0. \quad (5)$$

The excess demand function for the asset when  $\mu=1$ ,  $\beta=9/10$ , and  $\underline{y}=1$ :  $z_1-z_0$ , exhibited in Figure2, ascertains *Proposition 1*.

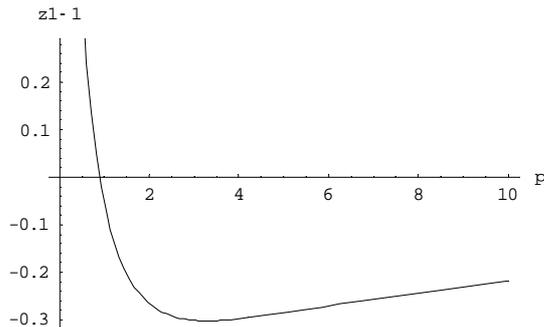


Figure2. Excess Demand Function (Exponential Function Case I)

Note, however, that this figure reveals an important property of the exponential utility function: when the asset price,  $p$ , is large, a further increase of  $p$  raises the demand for the asset: *i.e.* the asset as a *Giffen* good, or strong *income* effect.

### 2.3. CES Function

In (1-1) and (3-1), the separability of utility function is assumed. The following CES (Constant Elasticity of Substitution) type function does not satisfy the separability, where  $n$  is the degree of homogeneity and  $\tau$  is the elasticity of substitution.

$$U(c_1, c_2)=(c_1^{-\tau}+\beta c_2^{-\tau})^{-n/\tau} \quad (6)$$

When  $\tau=-1/2$ ,  $\beta=9/10$ , and  $n=1$ , the solution is  $p^*=\beta\underline{y}$ , as shown in Figure3.

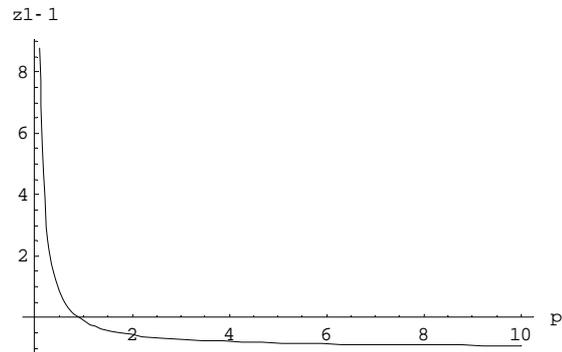


Figure3. Excess Demand Function (CES Function Case with  $\tau=-1/2$ )

When  $\tau=1/2$ ,  $\beta=9/10$ , and  $n=1$ , we have essentially the same excess demand function as in Figure 3.

### 2.4. Quadratic Function I

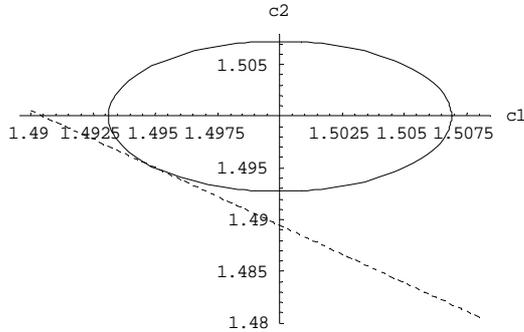
As pointed out in Introduction of this paper, the quadratic utility function has been utilized for convenience, since it allows the expression of expected utility as a function of mean and variance. This function, however, satisfies only one of the two conditions:  $u''(c)<0$ . For example,

$$u(c)=-c^2+3c+1 \quad (7)$$

has  $u'(c)<0$  when  $c>3/2$ . Although this does not prevent the maximizing behavior of the investors, the asset price is not determined when  $\underline{y}=3/2$ .

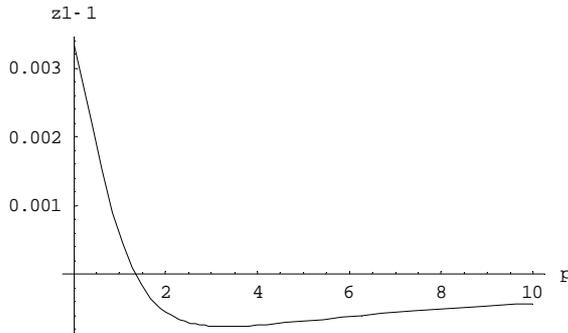
Only when  $\underline{y}<3/2$  *Proposition 1* holds. Indeed, we can show that the *certain* asset market equilibrium exists so long as  $\underline{y}<3/2$  holds, however  $\underline{y}$  is close to  $3/2$ . In order to show this, suppose that  $\underline{y}=299/200$ , and  $\beta=9/10$ . The attained maximum utility level,  $u_e$ , is computed as  $u_e=2469981/400000$ . In the following Figure 4, the indifference curve corresponding to  $u_e$  is depicted as the solid circle, while the budget line

passing through  $\{y, y\}$  is depicted as the dashed line. Thus,  $\{y, y\}$  is the utility maximizing consumption point.



**Figure 4. Utility Maximization (Quadratic Function Case)**

This equilibrium asset price is stable, as shown by the following Figure 5.



**Figure 5. Excess Demand Function (Quadratic Function Case I)**

Figure 5 also reveals an important property of the quadratic utility function: when the asset price,  $p$ , is large, a further increase of  $p$  raises the demand for the asset. The asset is a *Giffen* good, or has the strong *income* effect.

### 3. UNCERTAIN ASSET DIVIDEND CASE: UNCERTAIN FUTURE PERIOD

In this section, the uncertainty of asset dividend is introduced. Suppose that through the modified economic circumstances, the dividend in the second period becomes uncertain. When the expected dividend in the second period is greater than the certain dividend in the first period, the asset price in the modified situation may well be higher than the one in the original circumstance. When the former is exactly the same as the latter, however, the asset price may be expected to be lower than the one in the original circumstance, due to the risk-averse assumption. In considering this problem, however, a remark is in order: *i.e.*

the *income effect*, which is clear in (1-1), might prevent this expectation. The rise of asset price raises the income of the aggregate household. The Giffen good might produce a peculiar result. The model in this section is formulated as follows. In the first period, the aggregate investor possesses the asset on the amount of  $z_0=1$ . When  $p$  is the asset price, the aggregate investor possesses the income on the amount of  $(y+p) z_0$  in the first period. The aggregate investor plans the optimum consumption over the two period:  $c_1$ ,  $c_{21}$ , and  $c_{22}$ , and the purchase of the asset,  $z_1$ , believing that the purchase of the asset on the amount of  $z_1$ , yields the two different probable consumption in the second period,  $c_{21}=y_1 z_1$  with probability  $\lambda_1$ , and  $c_{22}=y_2 z_1$  with probability  $\lambda_2$ , with  $\lambda_1 y_1 + \lambda_2 y_2 = y$ , where  $\lambda_1 + \lambda_2 = 1$ , while the asset is bequeathed in the second period. Assuming  $u(c)$  to be the utility function, the aggregate investor's behavior is expressed by the following maximization.

$$\begin{aligned} \max & u(c_1) + \beta \{ \lambda_1 u(c_{21}) + \lambda_2 u(c_{22}) \} \\ \text{s.t.} & c_1 + p z_1 \leq (y+p) z_0, z_0=1, \\ & c_{21} \leq y_1 z_1, c_{22} \leq y_2 z_1 \end{aligned} \quad (1-2)$$

From this maximization, we have the demand functions,  $c_1(p)$ ,  $c_{21}(p)$ ,  $c_{22}(p)$ , and  $z_1(p)$ . The *uncertain pure exchange* asset market equilibrium is defined by  $p^{**}$ , which satisfies

$$\begin{aligned} z_1(p^{**}) &= 1, c_1(p^{**}) = y, \text{ and } c_{21}(p^{**}) = y_1, \\ c_{22}(p^{**}) &= y_2 \end{aligned} \quad (2-2)$$

It is examined if the following holds.

$$p^{**} \leq p^* \quad (9)$$

We have the following result [Fukiharu (1994)].

**Proposition 2.** Suppose that in addition to (3-1), the following holds.

$$R(c) = -u''(c)c/u'(c) \leq 1 \text{ and } R'(c) \geq 0, (c > 0). \quad (3-2)$$

Then, there exists the *uncertain pure exchange* asset market equilibrium,  $p^{**}$ , which satisfies (9).

In the proof of *Proposition 2*, the following *Lemma* is crucial.

**Lemma.** Suppose that (3-1) and (3-2) are satisfied. Then, the following inequality holds.

$$\lambda_1 u'(y_1) y_1 + \lambda_2 u'(y_2) y_2 \leq u'(y) y.$$

In Proposition 2,  $R(c) = -u''(c)/u'(c)$  is called the *relative risk-aversion*. The power function, defined in (4) satisfies (3-1) and (3-2), since  $R(c) = 1 - \gamma$ . In what follows, simulations are conducted by specifying utility function.

### 3.1 Power Function II: Future Uncertainty Case

When  $\gamma = 1/2$ , for the power function, defined in (4), we can actually derive  $p^{**}$ .

$$p^{**} = \beta \underline{y} \{ (\lambda_1 \underline{y}_1^{1/2} + \lambda_2 \underline{y}_2^{1/2})^2 / \underline{y} \}^{1/2} \leq \beta \underline{y} = p^*$$

When  $\lambda_1 = \lambda_2 = 1/2$ ,  $\underline{y}_1 = 0$ ,  $\underline{y}_2 = 2$ , and  $\underline{y} = 1$ ,  $\beta = 9/10$ , the excess demand function is exhibited as in what follows: stable equilibrium.

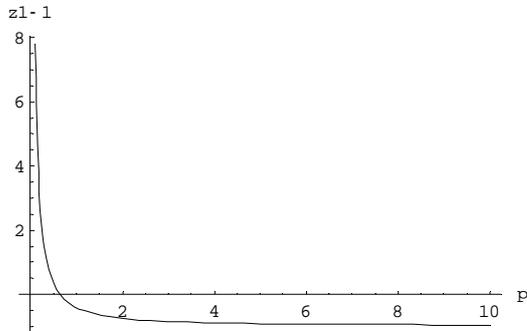


Figure 6. Excess Demand Function (Power Function II)

### 3.2. Exponential Function II Future Uncertainty Case

When the exponential function in (5) is assumed for the utility function,  $R(c) = \mu c$ , so that (3-2) is not satisfied. Note that when  $c$  is small, (3-2) is satisfied, while it is not the case when  $c$  is not small. In this subsection, we examine what would happen when the exponential function in (5) is assumed for the utility function. In what follows, we examine two cases, depending on the value of  $\underline{y}$ . We start with the case, in which  $\underline{y}$  is small.

#### 3.2.1 When $\underline{y}$ Is Small.

Suppose that

$$\lambda_1 = \lambda_2 = 1/2, \underline{y}_1 = 0, \underline{y}_2 = 2, \underline{y} = 1, \beta = 9/10 \quad (10-1)$$

When (10-1) is assumed,  $p^{**}$  is computed as

$$p^{**} = 9/(10e) = 0.331091 < 9/10 = \beta \underline{y} = p^*$$

Thus, (9) holds.

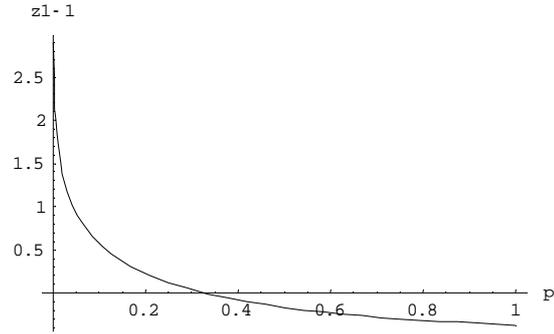


Figure 7. Excess Demand Function (Exponential Function IIA When  $\underline{y}$  Is Small)

The excess demand function is depicted in Figure 7, and  $p^{**} = 9/(10e)$  is the stable equilibrium.

#### 3.2.2. When $\underline{y}$ Is Not Small.

Suppose that

$$\lambda_1 = \lambda_2 = 1/2, \underline{y}_1 = 4, \underline{y}_2 = 6, \underline{y} = 5, \beta = 9/10 \quad (10-2)$$

When (10-2) is assumed,  $p^{**}$  is computed as

$$p^{**} = 9(3 + 2e^2)/(10e) = 5.88618 > 4.5 = \beta \underline{y} = p^*$$

Thus, (9) does not hold. This peculiar result is indeed derived by the household's maximizing behavior, as is shown by the following Figure 8, in which the indifference curve corresponding to the maximum *expected* utility level,  $u_{00} = 1.8839$  under the budget constraint on the second period, is depicted as the *solid* curve, while the budget line of the first period is depicted as the *dashed* curve.

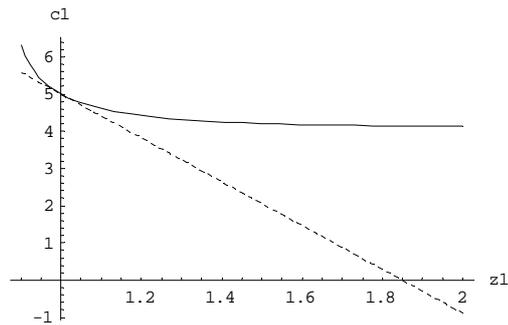
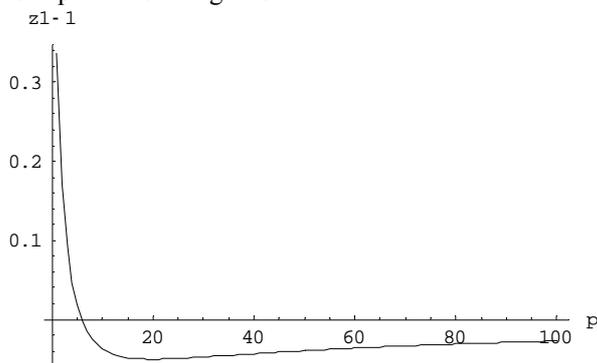


Figure 8. Household's Maximization of Expected Utility (Exponential Function IIB When  $\underline{y}$  Is Not Small)

The indifference curve is indeed *convex* to the origin, and we may safely conclude that  $\{1, 5\}$  is indeed the *expected* utility maximization and  $p^{**}$  is the *uncertain pure exchange* asset market equilibrium. Finally, it is examined if the equilibrium is stable. Utilizing the Newton

method, the excess demand function for the asset is depicted as in Figure 9.



**Figure 9. Excess Demand Function (Exponential Function IIB When  $\underline{y}$  Is Not Small)**

The *uncertain pure exchange* asset market equilibrium,  $p^{**} = 9(3+2e^2)/(10e)$  is stable. This figure reveals an important property of the exponential utility function: when the asset price,  $p$ , is large: a further increase of  $p$  raises the demand for the asset, in other words, the asset is a *Giffen* good, or has strong *income* effect.

### 3.3 Quadratic Function II: Future Uncertainty Case

In this subsection, we assume the quadratic utility function, (7). Note that the quadratic utility function does not satisfy (3-2), since  $R(c) = 2c/(3-2c)$ .

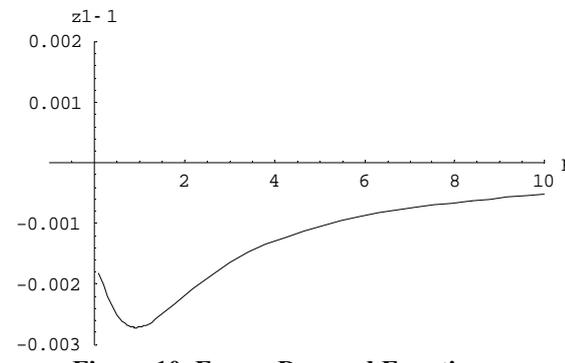
In the previous section, it was shown that so long as  $\underline{y} < 3/2$  is satisfied, there exists *certain* asset market equilibrium. It is shown in this section, even if  $\underline{y} < 3/2$  is satisfied, there is a possibility that exists no *uncertain* asset market equilibrium. To show this, suppose that parameters are specified by the following.

$$\lambda_1 = \lambda_2 = 1/2, \quad \underline{y}_1 = 3/2 - 11/100, \quad \underline{y}_2 = 3/2 + 10/100, \\ \underline{y} = 299/200, \quad \beta = 9/10 \quad (10-3)$$

When (10-3) is specified, we obtain that  $p^{**} = -639/1000$ . In other words, there exists no *uncertain* asset equilibrium.

In this *non-existence* case, some might expect the bubble: the expansion of asset price. This expectation is not supported. To show this, suppose that when there is no uncertainty, the equilibrium asset price,  $p^* = \beta \underline{y}$ , prevails where  $\underline{y} = 299/200 < 3/2$ . Now the uncertainty emerges where  $\underline{y}_1 = 3/2 - 11/100 < 3/2$ ,  $\underline{y}_2 = 3/2 + 10/100 > 3/2$ , and  $\lambda_1 \underline{y}_1 + \lambda_2 \underline{y}_2 = \underline{y}$ . In this uncertain circumstance, the excess demand is negative around the previous equilibrium price,  $p^* = \beta \underline{y}$ , as shown in

Figure 10.



**Figure 10. Excess Demand Function (Quadratic Function II)**

Thus, the asset price continuously declines, not rises. In other words, this analysis might be used for the *collapse of the bubble*. As the certain dividend increases in the bubble economy, the asset price rises. Approaching the satiation point of consumption, however, when the uncertainty emerges regarding the dividend, the bubble might collapse even if the expected dividend is invariable.

Meanwhile, suppose that the following holds.

$$\underline{y}_1 \leq \underline{y} \leq \underline{y}_2 < 3/2. \quad (11)$$

Under (11), it is shown that  $p^{**}$  is always positive, and  $p^{**} \leq p^*$  always holds. Stability of  $p^{**}$  is also guaranteed. Furthermore, under (11), we have the same type of excess demand function as in Figure 10. This figure reveals an important property of the quadratic utility function: when the asset price,  $p$ , is large, a further increase of  $p$  raises the demand for the asset. The asset is a *Giffen* good, or has the strong *income* effect.

### 4. UNCERTAIN ASSET DIVIDEND CASE: UNCERTAIN PRESENT AND FUTURE PERIODS

Finally, suppose that through the modified economic circumstances, the dividend in the first period also becomes uncertain, as well as in the second period. In the original circumstance, the dividend is certain with  $\underline{y}$  in both periods. In this section, it is assumed that in both periods, the dividends are  $\underline{y}_1$  with probability  $\lambda_1$  and  $\underline{y}_2$  with probability  $\lambda_2$ , where  $\lambda_1 \underline{y}_1 + \lambda_2 \underline{y}_2 = \underline{y}$ . Thus, the aggregate investor plans the optimum consumption over the two periods:  $c_{11}$ ,  $c_{12}$ ,  $c_{211}$ ,  $c_{212}$ ,  $c_{221}$ , and  $c_{222}$ , and the purchase of the asset,  $z_1$  and  $z_2$  in the first period, believing that the

purchase of the asset on the amount of  $z_1$  yields the two different probable consumption in the second period,  $c_{211}=\underline{y}_1 z_1$  with probability  $\lambda_1$ , and  $c_{212}=\underline{y}_2 z_1$  with probability  $\lambda_2$ , and the purchase of the asset on the amount of  $z_2$  yields the two different probable consumption in the second period,  $c_{221}=\underline{y}_1 z_2$  with probability  $\lambda_1$ , and  $c_{222}=\underline{y}_2 z_2$  with probability  $\lambda_2$ , with  $\lambda_1 \underline{y}_1 + \lambda_2 \underline{y}_2 = \underline{y}$ , where  $\lambda_1 + \lambda_2 = 1$ , while the asset is bequeathed in the second period. Assuming  $u(c)$  to be the utility function, the aggregate investor's behavior is expressed by the following maximization.

$$\begin{aligned} \max \quad & \lambda_1 u(c_{211}) + \lambda_2 u(c_{212}) + \beta \{ \lambda_1^2 u(c_{211}) + \lambda_1 \lambda_2 u(c_{212}) + \\ & \lambda_2 \lambda_1 u(c_{221}) + \lambda_2^2 u(c_{222}) \} \\ \text{s.t.} \quad & c_{11} + p_1 z_1 \leq (\underline{y}_1 + p_1) z_0, \quad c_{12} + p_2 z_2 \leq (\underline{y}_2 + p_2) z_0, \\ & z_0 = 1, \quad c_{211} \leq \underline{y}_1 z_1, \quad c_{212} \leq \underline{y}_2 z_1, \quad c_{221} \leq \underline{y}_1 z_2, \\ & c_{222} \leq \underline{y}_2 z_2. \end{aligned} \quad (1-3)$$

From this maximization, we have demand functions,  $c_{11}(p_1, p_2)$ ,  $c_{12}(p_1, p_2)$ ,  $c_{211}(p_1, p_2)$ ,  $c_{212}(p_1, p_2)$ ,  $c_{221}(p_1, p_2)$ ,  $c_{222}(p_1, p_2)$ ,  $z_1(p_1, p_2)$ , and  $z_2(p_1, p_2)$ . The *uncertain pure exchange* asset market equilibrium is defined by  $\{p_1^{**}, p_2^{**}\}$  which satisfies

$$\begin{aligned} z_1(p_1^{**}, p_2^{**}) = 1, \quad z_2(p_1^{**}, p_2^{**}) = 1, \quad c_{11}(p_1^{**}, p_2^{**}) = \underline{y}_1, \quad c_{12}(p_1^{**}, p_2^{**}) = \underline{y}_2, \\ c_{211}(p_1^{**}, p_2^{**}) = \underline{y}_1, \quad c_{212}(p_1^{**}, p_2^{**}) = \underline{y}_2, \quad c_{221}(p_1^{**}, p_2^{**}) = \underline{y}_1, \quad c_{222}(p_1^{**}, p_2^{**}) = \underline{y}_2. \end{aligned} \quad (2-3)$$

We obtain the following new result.

**Proposition 3:** Suppose that (3-1) and (3-2) are satisfied, where  $\underline{y}_1 \leq \underline{y} \leq \underline{y}_2$  holds. Then, the following holds:

$$p_1^{**} \leq p^* = \beta \underline{y} \leq p_2^{**} \quad (12)$$

In the proof of *Proposition 3*, Lemma is crucial. This conclusion implies that not all the *uncertain* asset prices can exceed the *certain* asset price. By the direct computation of equilibrium asset prices, it is ascertained that for exponential utility function (12) holds. In Section 3, where the uncertainty emerges only in the second period, it was shown that the income effect might exceed the effect of risk-aversion, resulting in the example in which the asset price rises in spite of the emergence of uncertainty. In this section with further introduction of uncertainty, it was shown that the income effect couldn't exceed the effect of risk-aversion: at least one asset price is smaller than the certain asset price. Note, however, that *expected* asset price in the uncertain world is greater than the asset price in the certain world.

## 5. CONCLUSIONS

This paper examined the variation of asset price when the uncertainty emerges regarding the prospect for dividend receipt, assuming that the investors are risk-aversers. It is expected that in this modified situation the demand for asset declines, leading to the decline of asset price. Due to the income effect the expectation might prove to be wrong. In this paper, following Lucas' formulation, we examined if there is any utility function, which provides the case in which the asset price rises in spite of the emergence of uncertainty regarding the prospect for dividend receipt. Constructing a risk-aversers' two period maximization problem, it was shown that if the uncertainty is introduced for only one period, the exponential utility function provides the case mentioned above. It was shown, however, that when the uncertainty is introduced for two periods, the exponential utility function does not provide the case. Thus, when the uncertainty is not so strong, the income effect may raise the asset price in spite of the increased uncertainty. It was shown in this case that the asset is a *Giffen* good when the asset price is already high. Historically, when the bubble economy took place, the market-structural change preceded it. This paper pointed out a theoretical possibility that even if the investors are risk-aversers the bubble economy may emerge. In examining this problem, it was also found that quadratic utility function might explain the *collapse* of bubble economy.

## 6. REFERENCES

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(For the computation and simulation, see my homepage:  
<http://home.hiroshima-u.ac.jp/fukito/index.htm>)