The Travelling Salesman Problem in Maritime Surveillance – Techniques, Algorithms and Analysis

Marlow, D.O. [1]**, P. Kilby** [2] **and G.N. Mercer** [3]

[1] Air Operations Division, Defence Science and Technology Organisation, Fishermans Bend, Victoria, Australia
[2] National ICT Australia, Canberra, ACT, Australia
[3] School of Physical, Environmental and Mathematical Sciences, University of New South Wales at the Australian Defence Force Academy, Campbell, ACT, Australia
Email: david.marlow@dsto.defence.gov.au

Keywords: *TSP, maritime surveillance, target classification, nearest neighbour, 2-opt*

EXTENDED ABSTRACT

The Royal Australian Air Force (RAAF) conducts maritime surveillance operations in order to deter non-state threats such as terrorism or illegal fishing. RAAF aircraft search various areas of ocean in order to classify as many ships as possible in the shortest possible time. This resembles the traditional Travelling Salesman Problem (TSP) but with many interesting variations – the "cities" (ships) are moving, time windows and precedence constraints are present, a total route length limit is enforced and the position of ships is usually only discovered as the route is flown. While the TSP itself is well-studied, these variations are not, particularly when grouped together as is the case with maritime surveillance.

This problem was initially presented at the 2007 Mathematics-in-Industry Study Group (MISG) in Wollongong, Australia to improve on a search method used by the Defence Science and Technology Organisation (DSTO) in modelling maritime surveillance. The DSTO model assumes that the ships are stationary due to the speed discrepancy between search aircraft and ships. MISG delegates chose to explore the problem as an application of the TSP. In this paper, analysis is undertaken that compares a nearest neighbour (NN) procedure (which best reflects that method used by aircrew) with three other techniques based on the more robust 2-opt method.

The parameter space is very large for this problem and several simplifying assumptions are made. In this paper, the Area of Interest (AI) size, aircraft speed and detection range is held constant, while ship speed and ship (target) numbers are varied. In total, 100 runs are generated for each case.

Results indicate that the percentage of targets classified steadily increases to a maximum and then steadily decreases as target density increases, for all cases of ship speed and method used. As target numbers increase, more targets are detected and classification rate increases. This maximum is close to 100% if the ships are not moving, down to 50% if all ships are travelling at 30 knots. Maximum flight time is the dominating constraint. At this threshold, there are too many targets to classify in the time available and the classification rate decreases.

For the input data and cases considered here, using a NN technique to solve the maritime surveillance TSP is only reasonable for cases where the number of targets in the AI is small (eg, up to around 20) or when all ships are moving at very fast speeds. However, if the number of targets is beyond around 50, a solution method based on 2-opt generally gives results that are around 10 percentage points better based on the percentage of targets classified for realistic operational scenarios. Using a 2-opt technique also produces more efficient searches for target numbers between 20-100, with completion times up to an hour less that for NN. A comparison of solution run times shows that NN is substantially faster, while the 2-opt "stationary ships" method appears better suited to larger target numbers as it has the shortest computation time of the chosen 2-opt variants.

Balancing the three Measures of Effectiveness of percentage of targets classified, mission time and computation time, for the cases considered, NN is found to be least suitable, and either the standard 2-opt method or the 2-opt stationary ships variant appear to be the most suitable choice overall.

These results suggest that the stationary ship assumption in the current model has validity. They also indicate an operational efficiency increase is achievable if on-board assistance incorporating a technique superior to NN (such as 2-opt) is provided to aircrew.

1. INTRODUCTION

Conducting maritime surveillance is an ongoing concern of the Australian Government and is a mandated role for the Australian Defence Force (ADF) under the 2000 Defence White Paper. Various platforms, particularly Royal Australian Air Force (RAAF) aircraft (such as AP-3C Orions), undertake maritime surveillance over Australia's northern approaches on a regular basis.

The aim in searching an Area of Interest (AI) is to find an optimal flight path for an aircraft such that it can classify the largest number of ships in the shortest possible time. This resembles the traditional and well-known Travelling Salesman Problem (TSP) in Operations Research (OR), where a "salesman" is required to find the shortest path that enables him to visit a number of cities once only and then return home.

Various exact techniques and heuristics have been developed to solve the TSP (eg, Gutin and Punnen (2002)). When applied to maritime surveillance however, there are interesting and complicated variations to consider. For example, the "cities" (ships) are moving with random velocities (hence it is a moving-target TSP), not all ship locations may be known to the aircrew in advance (so it is an "on-line" version of the TSP) and the aircraft has a finite fuel load (meaning that there is a "time window" on the search). Additionally, in this version the tours are "open" – the "salesman" (aircraft) does not have to return to the start position in the AI (rather, the aircraft will depart from and arrive at its home base).

While the TSP itself is well-studied, the variations considered here have a relatively recent history by comparison and are generally studied individually. For example, Helvig et al. (2003) considered various instances of the moving-target TSP including the issue of re-supply against some specific cases. Zhou et al. (2003) investigated moving cities and the addition/removal of cities and tested some evolutionary techniques against these cases. Larsen et al. (2004) examined the problem of time windows in a dynamic TSP both with and without *a priori* information and analysed whether waiting at specified "idle" points was more beneficial than waiting at the "current" location for a location's time window to open. Jiang et al. (2005) examined a situation applicable to maritime surveillance and tested two genetic algorithm (GA) techniques with different crossover methodologies against each other.

A recent paper by Grob (2006) is the most relevant direct comparison to the problem considered here. He describes a model used to simulate a scenario similar to that considered here and compares a Nearest Neighbour (NN) technique against an "n-k" heuristic (ie, consider a tour of edge length n and re-evaluate after k steps) for a standard case and with a ship prioritisation rule included. Similarities include the consideration of moving ships, a "cookie-cutter" radar assumption and the Measure of Effectiveness (MOE) of identifying targets. However, there are also differences in the assumptions used. Extra complexity considered here includes endurance limits to the aircraft and the inclusion of the on-line assumption. While Grob (2006) mentions both of these without including results, he does consider extra complexity not considered here, such as variable aircraft altitude, variable ship speed and heading during the aircraft's flight, and attaching priority scores to the ships according to ship type.

This problem was initially presented at the Mathematics-in-Industry Study Group (MISG) held at the University of Wollongong in Wollongong, New South Wales, Australia from 5-9 February 2007. The aim in presenting the problem at the MISG was to seek assistance in the computer modelling of search techniques in maritime surveillance. DSTO uses such modelling to conduct OR. The current methodology employed in the model uses a simple Genetic Algorithm (GA) technique but assumes that the ships are stationary. Delegates at the MISG chose to explore the problem as an application of the TSP. Kilby et al. (2007) describe the outcomes from the MISG, and this paper extends the ideas generated and work conducted during that event.

2. PROBLEM DESCRIPTION

2.1. Scenario

An indicative diagram of the scenario typical of maritime surveillance barrier patrols (Wagner et. al. (1999)) is given in Figure 1. The AI is represented as a square, although the AI shape is variable. Ships are represented by small triangles and move with random velocities. The aircraft radar detection range is indicated by the dashed circle. Waypoints are denoted by solid small circles and the default flight path (the minimum distance) by solid lines joining the waypoints.

Maritime surveillance requires classification (to the level of ship type) of all ships within the AI. The search spacing is pre-briefed and is based on the expected radar detection range for the particular ship type of interest in that scenario. The aircraft maintains a list of ship contacts or targets (priority target list) that need to be flown towards to be classified.

The aircraft must fly to targets not yet classified and will deviate from the default flight path to fly towards these. The priority target list changes as tracks move in and out of radar detection range and as targets are classified by the aircraft.

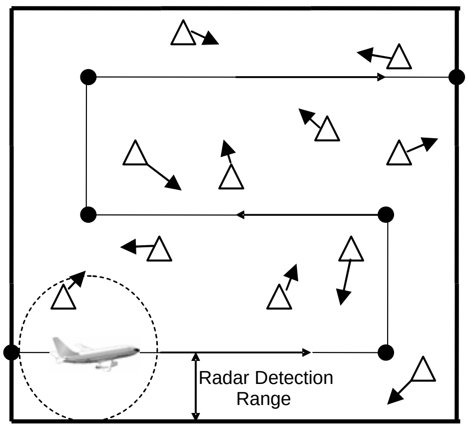

Figure 1: The maritime surveillance scenario

2.2. Assumptions and Simplifications

This problem has a large parameter space, so many assumptions have been made in order to simplify the problem in the first instance. The following effects are ignored in this paper, but are expected to be addressed in later work:

- Variation of aircraft altitude.
- Variation of ship speed.
- Impact of turning circles on tour length.
- Target prioritisation.
- Target clustering (eg, at fishing grounds).
- Impact of *a priori* third-party information on target locations in AI (eg, from satellites or other aircraft).

A simple "cookie-cutter" radar is used – if a ship is within radar detection (or classification) range, it is detected (or classified), else it is not. In reality, ship detection (involving forming and maintaining a track) and classification (determining target type) are not simple tasks – eg, the ability to classify a target may be affected by sea states.

2.3. Inputs

There is a range of potential inputs. They are:

- Surveillance aircraft speeds (100-350 kn). Aircraft used range from rotary-wing to high-altitude unmanned aircraft.
- Surveillance aircraft radar detection range (0-100 n mile). This can vary depending on the environment and target type.
- Surveillance aircraft classification range (0-20 n mile). This can vary depending on sensor performance and environmental conditions for a particular mission.
- AI size (100*100 to 300*300 n mile^2). While a square shape is used here, the AI can be any shape within these limits.
- Number of ships (0-1000 in real life). This can also vary according to seasonal factors and within a scenario as tracks are generated, or as they exit or enter the AI.
- Maximum flight time (variable). This is mainly of concern for crewed missions. Unmanned aircraft can fly for more than 24 hours at a time.
- Individual ship speed (0-30 kn). Most ships encountered will generally be travelling at up to 10 kn.
- Individual ship direction (0-360°).
- Presence of additional information on ship disposition (eg, from satellites).
- Waypoint position.

In military aviation and marine navigation, nautical miles and knots are used as the default units for speed and distance rather than SI units. Given that this work has a Defence origin, these units will be used throughout. The accepted abbreviations are n mile and kn respectively. In terms of SI units, 1 n mile is equal to 1852 m and 1 kn (ie, 1 n mile/hr) is equal to 0.514 ms^{-1}.

2.4. Constraints

There are two primary constraints:

1. The aircraft must remain inside the AI.
2. The aircraft must visit waypoints in order.

These are designed to keep the aircraft from straying too far from the search path. The first one is a modelling constraint – in reality, aircrew can decide to chase a target outside the AI if it does not adversely affect the mission (eg, the aircraft will not run out of fuel in doing so).

2.5. Measures of Effectiveness (MOEs)

The MOEs are:

- The percentage of targets classified.
- The time taken for the AI to be traversed.
- Solution run time.

A challenge of this problem type is balancing MOEs. One algorithm may classify every target but take a week of flying time; another may follow the shortest path and classify nothing; both are impractical. When modelling the mission, an algorithm may find the "perfect mix" of target classifications and flight time, but if it uses too much computational time it is also ineffective.

3. SOLUTION METHOD

The existing search technique used in the DSTO computational model is determined using a simple GA. A single-swap crossover with no mutations is used. The ships are assumed to be stationary, so the aircraft flies to intercept the next ship at its last known position. These positions are only updated when an "event" occurs, such as the detection of a new target. The validity of this assumption has not been tested.

The current search technique used by aircrew is essentially a NN search, as no software is currently provided to assist on operations. NN is considered effective in AIs with a low ship density. However, being a greedy algorithm, it is mathematically sub-optimal by nature. It is expected that effectiveness will reduce in a higher-density environment.

A simple alternative to NN is the 2-opt algorithm introduced by Croes (1958). This technique removes two segments of an existing path and forms a new path with the remaining segments (thus involving a change of path direction down one of the segments). If this path is shorter than the original, it is kept, else it is rejected. The process repeats until the shortest path is found. The method called "2-opt" here also uses the Or-opt method from Or (1976), where each tour segment of 5 consecutive visits is removed, and the cost of re-inserting it between every remaining pair of visits is calculated and replaced in the cheapest spot. Each segment of 4, 3, 2, and then 1 visit is then similarly tested.

Since NN looks only at the next visit, it is not greatly affected by ship movement. Because 2-opt plans a tour through all ships and the ships are moving, each 2-opt (and Or-opt) change requires a new intercept point to be calculated. The standard 2-opt version does this recalculation for every potential change. This can be quite expensive, so a "stationary ships" variant of 2-opt used here effectively ignores the effect of ship movement, updating when an event occurs as in the current DSTO model. The "jumping ships" variant of 2-opt also used here lies between. It calculates the best route treating the ships as stationary. It then recalculates intercept points for the new order, and repeats until the tour converges. If it does not converge after 10 iterations, the best one is kept.

4. RESULTS AND ANALYSIS

In this section, comparisons are made between the NN technique and the three 2-opt variants described previously. The aircraft follows a path indicated by the diagram in Figure 1. As well as the constraints, an additional heuristic is included that ensures that the aircraft does not chase ships too far from its current segment if it determines that it will catch it at a later stage of the journey.

4.1. Input Values

The constant values used in the model are:

- AI size: 300 * 300 n mile^2
- Aircraft speed: 300 kn
- Detection range: 50 n mile
- Classification range: 0 n mile (so the aircraft must fly up to a ship to classify it)
- Maximum time permitted in AI: 8 hr

The parameters that are varied in the results are:

- Solution method: NN, 2-opt, 2-opt stationary ships, 2-opt jumping ships
- Ship speed: 0, 5, 10, 20, 30 kn
- Number of targets in AI: seven to nine values ranging from 10 to 200

For each solution method, average values are taken across 100 datasets, so the number of cases run for each method is 5*(7 to 9)*100 = (3500 to 4500). Results are presented using actual numbers rather than in dimensionless terms (eg, ratio of aircraft speed to ship speed), as preliminary work indicates that the results do not scale (eg, higher ship speeds mean more ships enter the AI during the mission, thus affecting the percentage of classified targets).

4.2. Comparing Ship Speeds

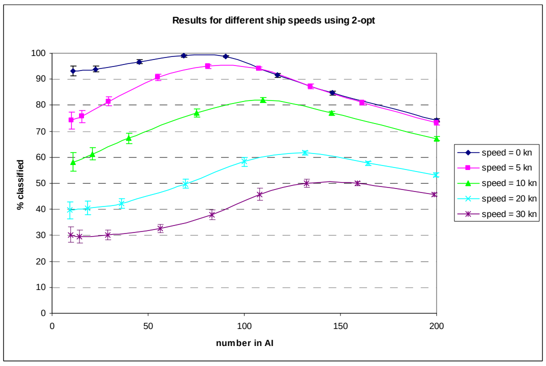

Figure 2. Classification rates using 2-opt for various ship speeds

Figure 2 shows the results across a range of target speeds for the 2-opt method. Slower ship speeds result in more classifications, with a maximum for stationary ships of almost 100%, decreasing through to the maximum for ships travelling at 30 kn of around 50%. For all ship speeds, the results show a steady increase in classifications to a local maximum as the numbers of targets in the AI increases, followed by a steady decrease. Error bars indicating the 95% confidence interval about the mean using a *t*-test are also shown. The largest variations are when ship numbers are small and ship speed is large, when variability between scenarios is likely to be greatest.

The initial increase can be explained in terms of detection ranges and ship density. If the number of targets is small, the aircraft flies through the AI rapidly and will finish well inside the maximum flight time of 8 hours. If only a handful of ships is present in the AI, the flight time will be close to the "default" flight time of 3 hours, as there will be little variation from the flight path to chase ships. If ship speed is large, the aircraft is more likely to miss ships that enter the AI after it has completed searching a particular area. As the number (and thus density) of targets increases, the aircraft is likely to detect (and classify) more targets. In turn, it will be "drawn" towards other ships which it might otherwise have missed if the ship density was lower. An increased detection range leads to more detections and the likelihood of more classifications (Mercer et al. (2007)).

The final decrease can be explained in terms of the available flight time. The local maximum in classifications is reached around the time that the aircraft flight time approaches the limit of 8 hours. Beyond that point, the number of targets is so large that it becomes impossible for the aircraft to classify them all in the available time.

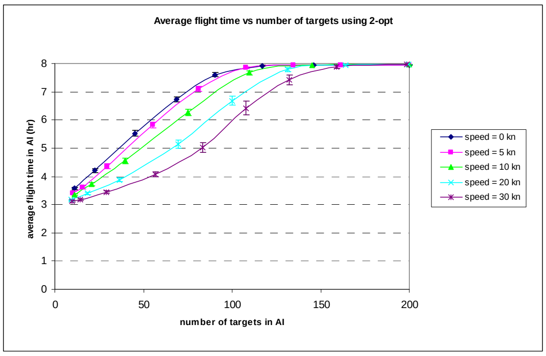

Figure 3. Flight time in AI versus number of targets in AI for 2-opt method

Figure 3 shows how mission flight time varies with number of targets for the 2-opt results and shows the link with maximum classification range in Figure 2. Maximum flight time is reached at around 100 targets for slower ship speeds, increasing to around 150 for speeds of 30 kn.

4.3. Comparing Solution Techniques

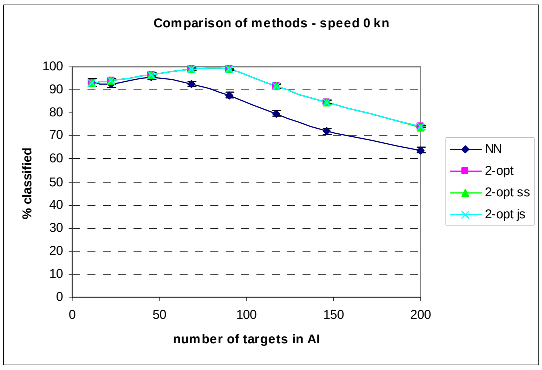

Figure 4. Comparison of NN and 2-opt techniques for stationary ships

Figure 4 shows a comparison between NN and the 2-opt methods for the case where all ships are stationary. If the number of ships in the AI is up to 50 targets, NN compares well with the 2-opt techniques. Between 50 and 100 targets, however, the difference in percentage of targets classified opens up to around 10 percentage points. Beyond 100 targets, the impact of maximum flight time begins, so the classification rate falls for all methods, but the difference of around 10 is maintained. The three techniques based on 2-opt provide almost identical results. The 95% confidence interval results show greatest variability in the instances where ship numbers are lowest, becoming insignificant when maximum flight time is reached.

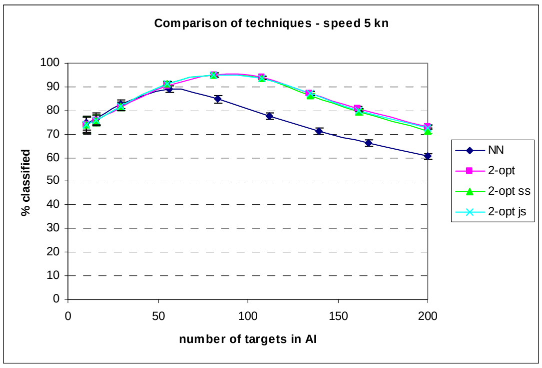

Figure 5. Comparison of NN and 2-opt techniques for ship speed 5 kn

Figure 5 shows the results when all ships are travelling at 5 kn, which may be deemed as the most "typical" real-life operational case. The results are similar to those shown in Figure 4, with the gap between the NN and 2-opt techniques again emerging at around 50 targets. The gap widens to around 15 percentage points in the percentage of targets classified and remains there beyond the maximum time threshold at around 100 targets until the 200 target maximum. The results for the three 2-opt techniques are identical until 200 targets, when the stationary ships method is slightly worse. The 95% confidence intervals are ±3 percentage points from the mean for lower numbers of targets, reducing to around ±1 at higher numbers.

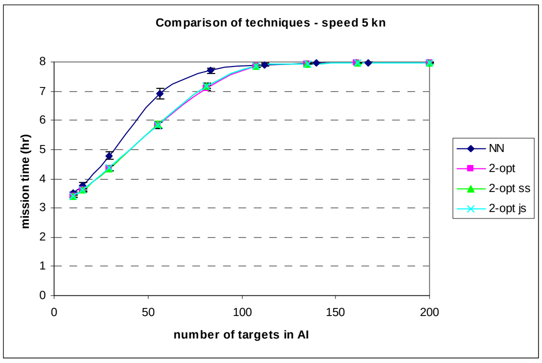

Figure 6. Comparison of mission time for various techniques, ship speed 5 kn

Figure 6 shows the comparison of mission times for the four solution techniques. An interesting observation is that while NN gives around the same percentage of classifications as 2-opt for up to 50 targets, this graph shows that it is less efficient in doing so, taking over an hour longer. The results for the stationary ship case are similar.

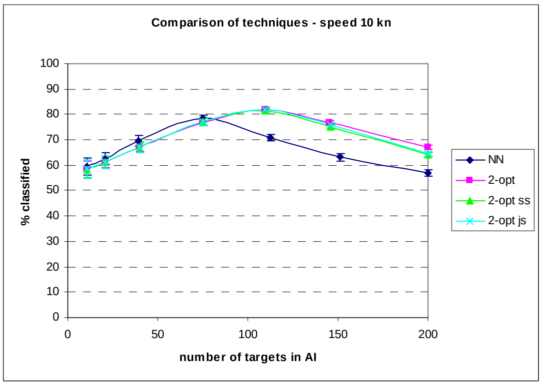

Figure 7. Comparison of NN and 2-opt techniques for ship speed 10 kn

The results for percentage of ships classified for ship speeds of 10 kn is shown in Figure 7. Although NN compares well with 2-opt up to around 75 targets, the 2-opt methods are superior beyond this number, with the standard 2-opt technique again slightly better than the others. The mission times at lower target numbers are similar to those for the 5 kn case.

Results for the cases where ship speeds are 20 kn and 30 kn are not shown, as these are the least realistic operationally. For the 20 kn case, it is noted that while 2-opt is still superior to NN, particularly when the number of targets exceeds 100, this advantage is reduced. For the 30 kn case, all methods provide almost identical results. The rapidly changing surface picture reduces the advantages of the 2-opt methods in these cases. In mission time, there is little difference between NN and 2-opt stationary ships in the 30 kn case, as the event-based nature of this 2-opt method struggles in the highly dynamic environment.

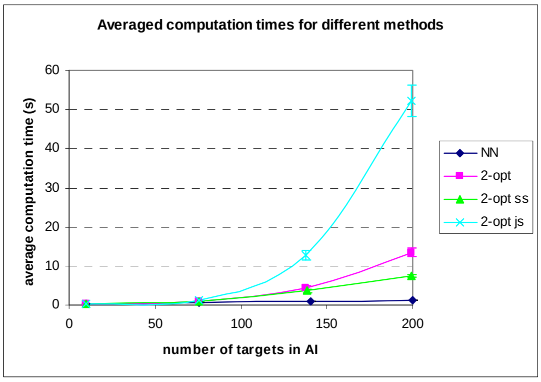

Figure 8. Comparison of computation time for different methods

The final graph in Figure 8 shows the run time for each method averaged over all speeds. Beyond 75 targets, the differences become apparent. As expected, NN is by far the quickest, followed by the 2-opt stationary ships method (which does not calculate the "moving" intercept point) and the 2-opt method (which does). The 2-opt jumping ships method is significantly worse than any other (due to its use of iteration to find the best route).

5. SUMMARY AND CONCLUSIONS

For the input data and cases considered here, using a NN technique to solve the maritime surveillance TSP is only reasonable for cases where the number of targets in the AI is small (eg, up to around 20) or when all ships are moving at very fast speeds. Otherwise, using a solution method based on 2-opt gives more efficient searches for between 20-100 targets, and better results (by around 10 percentage points) for the percentage of targets classified for beyond 50-75 targets. The 2-opt stationary ships method appears better suited to larger target numbers as it has the shortest computation time of the variants, followed by 2-opt and the significantly worse 2-opt jumping ships method. Combining these results, either the standard 2-opt or 2-opt stationary ships method is most suitable.

This paper has only presented a necessarily limited set of results for particular cases. Variations in detection ranges, maximum flight times and other parameters may yield different results. Future work will include a fuller exploration of the parameter space, progressively relaxing the assumptions and simplifications stated earlier. Planned examples are a study of the impact of aircraft turning circles and varying classification rates (Mercer et al. (2007)) on the various solution methods, and an analysis of “budgeting” criteria that force an aircraft to stay on schedule to complete the mission in the required time.

The initial implications of these results for the DSTO model is that using a “stationary ships” assumption may be acceptable. When considering the percentage of targets classified, the 2-opt stationary ships method gives results that are virtually identical to 2-opt through to 100 targets and are only 2-3% worse for higher target numbers. Computationally, it is superior to 2-opt at higher target numbers. The next step in this work will be to do a direct comparison between the GA method currently used in the model and the NN and 2-opt methods considered here.

Preliminary conclusions for maritime surveillance operations may also be drawn from these results. The generally superior results of 2-opt over NN for both target classification and mission time suggest on-board software to assist aircrew in conducting their search may be worth pursuing. However, any benefits (eg, in fuel savings, higher target classifications) would have to be weighed against the potential costs (eg, of integrating the radar picture with on-board software for real-time updates on all maritime patrol aircraft).

6. ACKNOWLEDGEMENTS

The authors would like to thank all those who worked on this problem at MISG 2007, particularly Dr Patrick Tobin, Ruth Luscombe, Dr Jason Looker, Dr Steve Barry and Roslyn Hickson. The authors also thank the MODSIM reviewers and internal DSTO reviewers for their comments.

7. REFERENCES

Croes, G.A. (1958), A method for solving traveling-salesman problems, *Operations Research*, 6, 791-812.

Grob, M.J.H.B. (2006), Routing of platforms in a maritime surface surveillance operation, *European Journal of Operational Research*, 170, 613-628.

Gutin, G. and A. Punnen (Eds) (2002), The Traveling Salesman Problems and its Variations, Kluwer Academic Publishers, Dordrecht.

Helvig, C.S., G. Robbins and A. Zelikovsky (2003), The moving-target traveling salesman problem, *Journal of Algorithms*, 49, 153-174.

Jiang, Q., R. Sarker and H. Abbass (2005), Tracking moving targets and the non-stationary traveling salesman problem, *Complexity International*, 11, 171-179.

Kilby, P., P. Tobin and R. Luscombe (2007), Moving target routing in maritime surveillance, *Proceedings of the 2007 Mathematics-in-Industry Study Group*, University of Wollongong, Wollongong, Australia, 5-9 February 2007.

Larsen, A., O.B.G. Madsen and M.M. Solomon (2004), The A Priori dynamic travelling salesman problem with time windows, *Transportation Science*, 38 (4), 459-472.

Mercer, G.N., S.I. Barry, D.O. Marlow and P. Kilby (2007), Investigating the effect of detection and classification range and aircraft dynamics on a simplified maritime surveillance scenario, *The ANZIAM Journal*, to appear.

Or, I. (1976), Travelling salesman-type combinatorial problems and their relation to the logistics of blood-banking, Ph.D. thesis, Northwest University, Evanston.

Wagner, D.H., W.C. Mylander and T.J. Sanders (editors) (1999), Naval Operations Analysis, Third Edition, Naval Institute Press, 421 pp., Annapolis.

Zhou, A., L. Kang and Z. Yan (2003), Solving dynamic TSP with evolutionary approach in real time, *Proceedings of the IEEE-CEC2003*, 2, 951-957.