Some Monte Carlo Evidence on the Hypothesis Testing for the Mean of the Stationary Vector Autoregressive Process

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Abstract: This paper deals with the hypothesis testing for the mean of the stationary vector autoregressive (VAR) process. We consider the situation in which a researcher's interest lies, not in detecting the lag order of the VAR model and/or in estimating coefficient matrices of the model, but in testing the hypothesis on the mean of the stationary VAR process. We investigate the finite sample performance of alternative testing procedures that are applicable in such situations. Two of these procedures are conventional methods, such as the Wald test statistic. This approach requires the determination of the lag order of the VAR model and estimation of coefficient matrices of the model, which are nuisance parameters for the researcher. The other one is the method which is recently developed by Kiefer, Vogelsang and Bunzel (2000). The comparative advantage of the approach is that it does not require such unnecessary inferences. On the other hand, the approach uses artificial accumulation of incorrectly specified OLS residuals, which could be costly in finite samples. Our aim is to provide some useful information for the researcher to select one of these procedures through Monte Carlo simulation experiments.

Key Words : *Hypothesis testing; Mean of stationary VAR process; Simulation experiments*

1. INTRODUCTION

In this paper, we deal with the hypothesis testing for the mean of the n-variate vector time series $\boldsymbol{y}_t = (y_{1t}, \dots, y_{nt})'$ which is generated by the following model:

$$\boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{u}_t, \quad \mathrm{E}(\boldsymbol{u}_t) = \boldsymbol{0}, \quad t = 1, \dots, T. \tag{1}$$

Then, the hypothesis for the mean of the vector time series, $\boldsymbol{y}_t$, can be represented as: $H_0 : \boldsymbol{R\mu} = \boldsymbol{r}$ and $H_1 : \boldsymbol{R\mu} \neq \boldsymbol{r}$, where $\boldsymbol{R}$ is an $m \times n$ full-row-rank-matrix and $\boldsymbol{r}$ is an m-dimensional column vector. For an economic example of $\boldsymbol{y}_t$, $\boldsymbol{R}$, and $\boldsymbol{r}$, suppose that $\boldsymbol{y}_t$ is constructed by two stock index returns from two different stock markets, then the hypothesis whether or not the means of these stock returns are equal can be expressed by $\boldsymbol{R} = (1, -1)$ and $\boldsymbol{r} = 0$.

In this paper we suppose that $\boldsymbol{u}_t$ in (1) is generated by the following vector autoregressive (VAR) model:

$$\boldsymbol{\Psi}(L)\boldsymbol{u}_t = \boldsymbol{\epsilon}_t, \tag{2}$$

where $\boldsymbol{\Psi}(L) = \boldsymbol{I}_n - \boldsymbol{\Psi}_1 L - \cdots - \boldsymbol{\Psi}_p L^p$ and all roots of $|\boldsymbol{\Psi}(z)| = 0$ are outside the unit circle and L denotes the lag operator. We further suppose that $\boldsymbol{\epsilon}_t$ is an independently and identically distributed (iid) sequence with mean $\boldsymbol{0}$, finite fourth moments, and $\mathrm{E}(\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t') = \boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}$ is a positive definite matrix.

Under the assumptions above, the vector time series, $\boldsymbol{y}_t$, is a stationary vector autoregressive process of order p with mean $\boldsymbol{\mu}$. The hypothesis testing can be performed with conventional test statistics such as the Wald test statistic, which uses ordinary least squares (OLS) estimators for the coefficient matrices of the following p-th order VAR model:

$$\boldsymbol{y}_t = \boldsymbol{\mu}^* + \boldsymbol{\Psi}_1\boldsymbol{y}_{t-1} + \cdots + \boldsymbol{\Psi}_p\boldsymbol{y}_{t-p} + \boldsymbol{\epsilon}_t, \tag{3}$$

where, $\boldsymbol{\mu}^* = (\boldsymbol{I}_n - \boldsymbol{\Psi}_1 - \cdots - \boldsymbol{\Psi}_p)^{-1}\boldsymbol{\mu}$. However, this method requires the determination of the lag order of the VAR model as well as OLS estimators of the coefficient matrices of the VAR model (3), which are nuisance parameters for the researcher because his or her interest lies in testing the hypothesis for the mean of $\boldsymbol{y}_t$. These requirements could be considered as drawbacks of the procedure.

Recently, Kiefer, Vogelsang and Bunzel (2000) proposed a new approach, which is attractive because it does not require the determination of the lag length and/or the estimation of the nuisance coefficient matrices. However, this method uses a partial sum process of OLS residuals from a misspecified model, which could be costly in finite samples. In this paper, we attempt to provide information as to which procedure is more appropriate for the researcher, through the use of Monte Carlo simulation experiments. This paper is organized as follows. In Section 2, we introduce two conventional methods and the procedure of Kiefer, Vogelsang and Bunzel (2000). In Section 3, we show some simulation results. Our conclusion is provided in Section 4.

2. ALTERNATIVE PROCEDURES

2.1. Conventional Procedures

We define $\hat{\boldsymbol{\mu}} = (\boldsymbol{I}_n - \hat{\boldsymbol{\Psi}}_1 - \cdots - \hat{\boldsymbol{\Psi}}_p)^{-1}\hat{\boldsymbol{\mu}}^*$, where $(\hat{\boldsymbol{\mu}}^*, \hat{\boldsymbol{\Psi}}_1, \cdots, \hat{\boldsymbol{\Psi}}_p)$ is the matrix of OLS estimators for $(\boldsymbol{\mu}^*, \boldsymbol{\Psi}_1, \cdots, \boldsymbol{\Psi}_p)$. From Lütkepohl (1993, p.77), $T^*(\hat{\boldsymbol{\mu}} - \boldsymbol{\mu}) \Rightarrow N(\boldsymbol{0}, \boldsymbol{\Omega})$, where " $\Rightarrow$" denotes weak convergence, $T^* = (T - p)$, and $\boldsymbol{\Omega} = (\boldsymbol{I}_n - \boldsymbol{\Psi}_1 - \cdots - \boldsymbol{\Psi}_p)^{-1}\boldsymbol{\Sigma}(\boldsymbol{I}_n - \boldsymbol{\Psi}_1 - \cdots - \boldsymbol{\Psi}_p)^{-1\prime}$. Then, under H_0, $T^{*1/2}(\boldsymbol{R}\hat{\boldsymbol{\mu}} - \boldsymbol{r}) \Rightarrow N(\boldsymbol{0}, \boldsymbol{R\Omega R}')$, and hence the asymptotic null distribution of the following W_1 test statistic is χ^2 distribution with m degrees of freedom:

$$W_1 = T^*(\boldsymbol{R}\hat{\boldsymbol{\mu}} - \boldsymbol{r})'(\boldsymbol{R}\hat{\boldsymbol{\Omega}}\boldsymbol{R}')^{-1}(\boldsymbol{R}\hat{\boldsymbol{\mu}} - \boldsymbol{r}), \qquad (4)$$

where $\hat{\boldsymbol{\Omega}} = (\boldsymbol{I}_n - \hat{\boldsymbol{\Psi}}_1 - \cdots - \hat{\boldsymbol{\Psi}}_p)^{-1}\hat{\boldsymbol{\Sigma}}(\boldsymbol{I}_n - \hat{\boldsymbol{\Psi}}_1 - \cdots - \hat{\boldsymbol{\Psi}}_p)^{-1\prime}$. Here, $\hat{\boldsymbol{\Sigma}} = T^{*-1}\sum_{t=(p+1)}^{T}\hat{\boldsymbol{\epsilon}}_t\hat{\boldsymbol{\epsilon}}_t'$ with $\hat{\boldsymbol{\epsilon}}_t$ being the OLS residuals from (3) for $t = (p+1), \ldots, T$.

In addition, from Lütkepohl (1993, p.77), we see $T^{1/2}(\tilde{\boldsymbol{\mu}} - \boldsymbol{\mu}) \Rightarrow N(\boldsymbol{0}, \boldsymbol{\Omega})$, where $\tilde{\boldsymbol{\mu}} = T^{-1}\sum_{t=1}^{T}\boldsymbol{y}_t$. Hence, the asymptotic null distribution of the following W_2 test statistic is also χ^2 distribution with m degrees of freedom:

$$W_2 = T(\boldsymbol{R}\tilde{\boldsymbol{\mu}} - \boldsymbol{r})'(\boldsymbol{R}\hat{\boldsymbol{\Omega}}\boldsymbol{R}')^{-1}(\boldsymbol{R}\tilde{\boldsymbol{\mu}} - \boldsymbol{r}). \qquad (5)$$

2.2. An Alternative Procedure

In this subsection, we introduce the approach developed by Kiefer, Vogelsang and Bunzel (2000). (Although the multivariate regression model is not discussed in their paper, the extension is straightforward.) We define $\tilde{\boldsymbol{u}}_t = \boldsymbol{y}_t - \tilde{\boldsymbol{\mu}}$, and $\tilde{\boldsymbol{u}}_t$ can be expressed as $\boldsymbol{u}_t - (\tilde{\boldsymbol{\mu}} - \boldsymbol{\mu})$. Then, from the functional central limit theorem (FCLT) and the continuous mapping theorem, we see $T^{-1/2}\sum_{t=1}^{[Ts]}\tilde{\boldsymbol{u}}_t \Rightarrow \boldsymbol{\Lambda V}_n(s)$, where $\boldsymbol{V}_n(s) = \boldsymbol{W}_n(s) - s\boldsymbol{W}_n(1)$ and $\boldsymbol{\Lambda}$ is a matrix such that $\boldsymbol{\Omega} = \boldsymbol{\Lambda\Lambda}'$. Here, $\boldsymbol{W}_n(s)$ denotes an n-dimensional standard Brownian motion and $[Ts]$ denotes the largest integer that is less than or equal to Ts where $s \in [0, 1]$. Consequently, with the definition of $\tilde{\boldsymbol{S}}_t = \sum_{j=1}^{t}\tilde{\boldsymbol{u}}_j$, we see that $\tilde{\boldsymbol{\Omega}} = T^{-2}\sum_{t=1}^{T}\tilde{\boldsymbol{S}}_t\tilde{\boldsymbol{S}}_t' \Rightarrow \boldsymbol{\Lambda}(\int_0^1\boldsymbol{V}_n(s)\boldsymbol{V}_n(s)'ds)\boldsymbol{\Lambda}'$. In addition, under H_0, $T^{1/2}(\boldsymbol{R}\tilde{\boldsymbol{\mu}} - \boldsymbol{r}) \Rightarrow \boldsymbol{R\Lambda W}_n(1)$. Thereby, under H_0:

$$\begin{aligned} Q &= T(\boldsymbol{R}\tilde{\boldsymbol{\mu}} - \boldsymbol{r})'(\boldsymbol{R}\tilde{\boldsymbol{\Omega}}\boldsymbol{R}')^{-1}(\boldsymbol{R}\tilde{\boldsymbol{\mu}} - \boldsymbol{r}) \\ &\Rightarrow \boldsymbol{W}_m(1)'(\textstyle\int_0^1\boldsymbol{V}_m(s)\boldsymbol{V}_m(s)'ds)^{-1}\boldsymbol{W}_m(1). \qquad (6) \end{aligned}$$

It is noteworthy that the asymptotic null distribution of the test statistic depends only on the number of restrictions, m, and that the distribution is nuisance-parameter free, although it is nonstandard. Because Kiefer, Vogelsang and Bunzel (2000) simulated and provided asymptotic critical values for $\boldsymbol{W}_m(1)'(\int_0^1\boldsymbol{V}_m(s)\boldsymbol{V}_m(s)'ds)^{-1}\boldsymbol{W}_m(1)/m$, we use the $F^*(\equiv Q/m)$ test statistic instead of the Q test statistic.

3. SIMULATION EXPERIMENTS

3.1. Lag Length Selection

Both the W_1 and W_2 test statistics require the determination of the autoregressive lag order p. We determine this using the Schwartz criterion (SC). We set the minimum lag as 1 and the maximum lag (p_{max}) as 8. The reasons why we apply SC to determine the lag length are: (i) it is consistent; and (ii) it is frequently used in empirical works. The SC for the lag order p, $SC(p)$, is calculated as:

$$SC(p) = \ln|\tilde{\boldsymbol{\Sigma}}| + \ln(T - p_{max})\frac{n^2 p}{T - p_{max}}, \qquad (7)$$

where $\tilde{\boldsymbol{\Sigma}} = (T - p_{max})^{-1}\sum_{t=(p_{max}+1)}^{T}\tilde{\boldsymbol{\epsilon}}_t\tilde{\boldsymbol{\epsilon}}_t'$ with $\tilde{\boldsymbol{\epsilon}}_t$ being the OLS residuals from (3) for $t = (p_{max}+1), \ldots, T$.

3.2. The Design for Simulation Experiments

We generated the n-variate time series $\boldsymbol{y}_t$ by the following DGP:

$$\boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{u}_t, \quad \boldsymbol{u}_t = \boldsymbol{\Psi}\boldsymbol{u}_{t-i} + \boldsymbol{\epsilon}_t \quad \boldsymbol{\epsilon}_t \sim \text{iid}N(\boldsymbol{0}, \boldsymbol{\Sigma}), \qquad (8)$$

where $i = 1, 6$ and $\boldsymbol{\Sigma}$ is an $n \times n$ positive definite matrix. For simplicity we assumed that $\boldsymbol{\Psi} = \psi\boldsymbol{I}_n$ and $\boldsymbol{\mu} = \delta\boldsymbol{\iota}$, where $\boldsymbol{\iota}$ is the n-dimensional column vector of ones. We set $\boldsymbol{R} = \boldsymbol{I}_n$ and $\boldsymbol{r} = \boldsymbol{0}$ in all cases. Consequently, the variance-covariance matrix of the innovation process in (8), $\boldsymbol{\Sigma}$, does not affect the value of the three test statistics W_1, W_2, and F^*, and thereby we set $\boldsymbol{\Sigma} = \boldsymbol{I}_n$ in our simulation experiments. For each Monte Carlo simulation, we generated 50000 series of length $(T + 100)$ from (8) and used the last T observations to calculate the test statistics. The (nominal) size of each test was always set equal to 0.05. All computations were performed using the GAUSS software with the RNDNS function.

First, we performed simulation experiments to examine the properties of the test statistics when H_0 holds, with the following parameter settings:

$$\delta = 0; \quad \psi = 0.8, 0.6, \ldots, -0.8; \quad n = 1, 2, 3; \\ T = 50, 100, 200, 400, \ldots, 3200. \qquad (9)$$

We tried the large sample sizes, such as $T = 1600$ and 3200, in order to observe how size distortions disappear as the sample size increases.

Table 1: Actual sizes of the test statistics

($n = 2$, $i = 1$)

T	$\psi =$	0.8	0.6	0.4	0.2	0.0	−0.2	−0.4	−0.6	−0.8
50	F^*	0.207	0.114	0.083	0.065	0.052	0.041	0.029	0.018	0.006
	$W_1(1)$	0.306	0.177	0.131	0.107	0.092	0.082	0.074	0.070	0.065
	$W_1(8)$	0.292	0.172	0.128	0.105	0.092	0.082	0.078	0.075	0.081
	$W_2(1)$	0.307	0.178	0.132	0.109	0.094	0.083	0.076	0.071	0.067
	$W_2(8)$	0.293	0.174	0.129	0.107	0.093	0.084	0.079	0.077	0.082
100	F^*	0.132	0.084	0.067	0.058	0.051	0.045	0.038	0.029	0.015
	$W_1(1)$	0.185	0.113	0.090	0.078	0.070	0.066	0.063	0.061	0.059
	$W_1(8)$	0.177	0.111	0.089	0.078	0.070	0.066	0.064	0.062	0.064
	$W_2(1)$	0.185	0.114	0.090	0.078	0.071	0.066	0.063	0.061	0.059
	$W_2(8)$	0.178	0.112	0.089	0.078	0.071	0.066	0.064	0.062	0.064
200	F^*	0.088	0.065	0.058	0.054	0.050	0.047	0.043	0.037	0.026
	$W_1(1)$	0.115	0.081	0.069	0.063	0.059	0.057	0.056	0.054	0.054
	$W_1(8)$	0.111	0.079	0.069	0.062	0.059	0.057	0.055	0.055	0.057
	$W_2(1)$	0.115	0.081	0.069	0.063	0.059	0.057	0.056	0.054	0.054
	$W_2(8)$	0.111	0.079	0.069	0.062	0.059	0.057	0.056	0.055	0.057
400	F^*	0.070	0.059	0.056	0.053	0.052	0.050	0.048	0.045	0.036
	$W_1(1)$	0.082	0.066	0.060	0.057	0.056	0.054	0.054	0.053	0.052
	$W_1(8)$	0.081	0.065	0.060	0.057	0.056	0.055	0.054	0.054	0.054
	$W_2(1)$	0.082	0.066	0.060	0.057	0.056	0.054	0.054	0.053	0.052
	$W_2(8)$	0.081	0.065	0.060	0.057	0.056	0.055	0.054	0.054	0.054
800	F^*	0.058	0.053	0.051	0.051	0.050	0.049	0.048	0.046	0.041
	$W_1(1)$	0.065	0.057	0.054	0.052	0.052	0.051	0.051	0.050	0.050
	$W_1(8)$	0.064	0.056	0.054	0.053	0.052	0.052	0.051	0.051	0.050
	$W_2(1)$	0.065	0.057	0.054	0.052	0.052	0.051	0.051	0.050	0.050
	$W_2(8)$	0.064	0.056	0.054	0.053	0.052	0.052	0.051	0.051	0.050
1600	F^*	0.054	0.051	0.050	0.050	0.050	0.049	0.049	0.048	0.046
	$W_1(1)$	0.056	0.053	0.052	0.051	0.051	0.051	0.050	0.050	0.050
	$W_1(8)$	0.056	0.053	0.052	0.052	0.051	0.051	0.051	0.051	0.051
	$W_2(1)$	0.056	0.053	0.052	0.051	0.051	0.051	0.050	0.050	0.050
	$W_2(8)$	0.056	0.053	0.052	0.052	0.051	0.051	0.051	0.051	0.051
3200	F^*	0.054	0.052	0.052	0.052	0.052	0.052	0.051	0.051	0.050
	$W_1(1)$	0.053	0.051	0.050	0.050	0.050	0.050	0.050	0.050	0.050
	$W_1(8)$	0.053	0.051	0.050	0.050	0.050	0.050	0.050	0.050	0.050
	$W_2(1)$	0.053	0.051	0.050	0.050	0.050	0.050	0.050	0.050	0.050
	$W_2(8)$	0.053	0.051	0.050	0.050	0.050	0.050	0.050	0.050	0.050

Note: Reported are rejection rates of the hypothesis $H_0 : \boldsymbol{\mu} = \mathbf{0}$ with 5% asymptotic critical values for the model (8) with $\boldsymbol{\epsilon}_t$ being a bivariate standard Gaussian white noise over 50000 replications. The numbers in parenthesis denote p_{max}.

Table 2: Actual sizes of the test statistics

($n = 2$, $i = 6$)

T	$\psi =$	0.8	0.6	0.4	0.2	0.0	−0.2	−0.4	−0.6	−0.8
50	F^*	0.508	0.301	0.172	0.097	0.052	0.023	0.009	0.002	0.000
	$W_1(4)$	0.693	0.496	0.321	0.185	0.092	0.038	0.012	0.003	0.001
	$W_2(4)$	0.799	0.529	0.329	0.188	0.093	0.039	0.014	0.007	0.009
100	F^*	0.421	0.225	0.133	0.084	0.051	0.029	0.014	0.005	0.001
	$W_1(4)$	0.719	0.495	0.304	0.163	0.070	0.023	0.005	0.000	0.000
	$W_2(4)$	0.783	0.509	0.306	0.163	0.071	0.023	0.005	0.001	0.000
200	F^*	0.277	0.146	0.095	0.069	0.050	0.035	0.023	0.012	0.002
	$W_1(4)$	0.721	0.483	0.290	0.146	0.059	0.016	0.003	0.000	0.000
	$W_2(4)$	0.756	0.488	0.291	0.146	0.059	0.016	0.003	0.000	0.000
400	F^*	0.168	0.098	0.074	0.061	0.052	0.043	0.034	0.022	0.008
	$W_1(4)$	0.719	0.474	0.285	0.142	0.056	0.014	0.002	0.000	0.000
	$W_2(4)$	0.738	0.477	0.285	0.142	0.056	0.014	0.002	0.000	0.000
800	F^*	0.108	0.072	0.060	0.054	0.050	0.045	0.040	0.033	0.019
	$W_1(4)$	0.718	0.478	0.283	0.139	0.052	0.012	0.001	0.000	0.000
	$W_2(4)$	0.727	0.479	0.283	0.139	0.052	0.012	0.001	0.000	0.000
1600	F^*	0.078	0.061	0.055	0.052	0.050	0.047	0.044	0.039	0.030
	$W_1(4)$	0.718	0.477	0.282	0.138	0.051	0.012	0.001	0.000	0.000
	$W_2(4)$	0.722	0.477	0.282	0.138	0.051	0.012	0.001	0.000	0.000
3200	F^*	0.065	0.057	0.054	0.053	0.052	0.051	0.049	0.046	0.039
	$W_1(4)$	0.718	0.477	0.278	0.137	0.050	0.011	0.001	0.000	0.000
	$W_2(4)$	0.720	0.477	0.278	0.137	0.050	0.011	0.001	0.000	0.000

Note: Reported are rejection rates of the hypothesis $H_0 : \boldsymbol{\mu} = \mathbf{0}$ with 5% asymptotic critical values for the model (8) with $\boldsymbol{\epsilon}_t$ being a bivariate standard Gaussian white noise over 50000 replications. The numbers in parenthesis denote p_{max}.

Table 3: Size-adjusted powers of the test statistics

($n = 2$, $i = 1$, $T = 100$)

ψ	$\delta =$	0.00	0.04	0.08	0.12	0.16	0.20	0.24	0.28
0.0	F^*	0.050	0.066	0.119	0.212	0.340	0.484	0.622	0.738
	$W_1(1)$	0.050	0.074	0.150	0.288	0.476	0.671	0.832	0.929
	$W_1(8)$	0.050	0.074	0.150	0.287	0.476	0.671	0.832	0.929
	$W_2(1)$	0.050	0.074	0.151	0.290	0.478	0.676	0.835	0.932
	$W_2(8)$	0.050	0.074	0.151	0.290	0.478	0.675	0.835	0.932
0.4	F^*	0.050	0.055	0.072	0.103	0.148	0.206	0.275	0.354
	$W_1(1)$	0.050	0.057	0.082	0.123	0.185	0.269	0.372	0.486
	$W_1(8)$	0.050	0.057	0.082	0.123	0.185	0.269	0.372	0.486
	$W_2(1)$	0.050	0.058	0.083	0.125	0.187	0.275	0.378	0.493
	$W_2(8)$	0.050	0.058	0.083	0.125	0.187	0.274	0.378	0.493
−0.4	F^*	0.050	0.083	0.194	0.379	0.585	0.756	0.870	0.934
	$W_1(1)$	0.050	0.098	0.260	0.527	0.788	0.938	0.989	0.999
	$W_1(8)$	0.050	0.098	0.260	0.527	0.788	0.937	0.989	0.999
	$W_2(1)$	0.050	0.099	0.261	0.529	0.789	0.939	0.989	0.999
	$W_2(8)$	0.050	0.099	0.261	0.529	0.789	0.939	0.989	0.999

Note: Reported are size-adjusted rejection rates of the hypothesis $H_0 : \boldsymbol{\mu} = \mathbf{0}$ with 5% asymptotic critical values for the model (8) with $\boldsymbol{\epsilon}_t$ being a bivariate standard Gaussian white noise over 50000 replications. The numbers in parenthesis denote p_{max}.

Second, we performed simulation experiments to examine the properties of the test statistics when H_0 does not hold, with the following parameter settings:

$$\delta = 0.00, 0.04, \ldots, 0.28; \quad \psi = 0.4, 0.0, -0.4;$$
$$i = 1; \quad n = 1, 2, 3; \quad T = 50, 100, 200. \quad (10)$$

3.3. Simulation Results

In this subsection, we report the simulation results. In order to save space, we only show selected results.

Table 1 tabulates the rejection rates for the case where $\delta = 0$, $i = 1$, and $n = 2$. When $i = 1$, since DGP is a VAR model of order one, the rejection rates for $W_1(1)$ and $W_2(1)$ are the results corresponding to the case where the correct lag length is known. From this table, we observe that when $0 \leq \psi$, F^* exceeds W_1 and W_2 in terms of size stability even if the correct lag length is known. For example, for the case where $T = 100$ and $\psi = 0.6$, the actual size of F^* is 0.084, while that of $W_1(1)$ is 0.113. On the other hand, when $\psi < 0$ and the sample sizes are relatively small, it is observed that F^* suffers from conservative size distortion, while W_1 and W_2 show relatively stable performances. For example, for the case where $T = 100$ and $\psi = -0.6$, the actual size of F^* is 0.029, while that of $W_1(1)$ is 0.061. Although F^* suffers from larger size distortions in absolute value, it is a conservative size distortion and so it is notable that when H_0 is rejected with F^*, the result is reliable.

Table 2 reports rejection rates for the case where $\delta = 0$, $i = 6$, and $n = 2$. We set $p_{max} = 4$ so that we can see how W_1 and W_2 perform when the lag length is underestimated. On the other hand, because F^* does not depend on p_{max}, at least in large samples, fairly good performance is expected. From this table, we can observe that, when size distortions are liberal, W_1 and W_2 suffer from severe size distortions even when $800 \leq T$, while the size distortions of F^* are small with such sample sizes. For example, in the case where $T = 1600$ and $\psi = 0.6$, the actual size of F^* is 0.061, while that of $W_j(4)$ is 0.477 for $j = 1, 2$. Even in small samples, F^* is much more stable than $W_1(4)$ and $W_2(4)$. For example, in the case where $T = 100$ and $\psi = 0.6$, the actual size of F^* is 0.225, while that of $W_1(4)$ is 0.495 and that of $W_2(4)$ is 0.509. When size distortions are conservative, it is observed that in most cases F^* excels $W_1(4)$ and $W_2(4)$. For example, in the case where $T = 200$ and $\psi = -0.4$, the actual size of F^* is 0.023, while that of $W_1(j)$ is 0.003 for $j = 1, 2$.

Table 3 shows size-adjusted rejection rates when δ increases from 0. From this table, we can observe that W_1 and W_2 are more powerful than F^* in all cases. For example, in the case where $T = 100$, $\psi = 0$, and $\delta = 0.12$, the size-adjusted rejection rate of F^* is 0.212, while that of $W_1(8)$ is 0.287.

4. CONCLUSION

In this paper, we have investigated the sampling performance of alternative testing procedures of the hypothesis testing for the mean of the stationary vector autoregressive (VAR) process. Two of these procedures are conventional methods, such as the Wald test statistic. The other one is the method which is recently developed by Kiefer, Vogelsang and Bunzel (2000).

Our findings from the Monte Carlo simulation experiments may be summarized as follows:

- (i) The size distortions of the F^* test statistic are smaller than those of the W_1 and W_2 test statistics when both tests have liberal size distortions.
- (ii) When p_{max} is less than the true lag length, W_1 and W_2 suffer from severe size distortions caused by mis-specification, while F^* generally shows good performance.
- (iii) There are some cases where F^* suffers from larger size distortions in absolute value, but they are only conservative, rather than liberal, size distortions.
- (iv) The size-adjusted powers of W_1 and W_2 are higher than those of F^*.

These results imply that there seems to be an unfortunate trade-off between the size and power properties, that is, F^* generally excels W_1 and W_2 in terms of the size stability, while W_1 and W_2 have higher power than F^*.

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6. REFERENCES

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