# On The Relationship Between Runoff Coefficient And Catchment Initial Conditions

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Abstract: The runoff coefficient, estimated as the ratio between quick flow and rainfall volume, on an event basis, has been analyzed in an empirical framework, as a function of the initial catchment state conditions prior to an event, such as pre-event soil moisture and pre-event base flow. The resulting relationships have been tested for their utility in runoff prediction, focusing on the variability of the catchment response, dependent on the initial condition itself. The variability of the catchment response is also addressed in a conceptually based stochastic model for stream flow simulation and forecasting. The watershed response to effective rainfall is considered as deriving from the response of linear reservoirs in parallel, representing contribution to stream flow from the fast (surface flow and shallow subsurface flow) and slow (deep subsurface flow and groundwater flow) components. The parameter that indicates how the hydrograph is split between these components, responsible for the non linearity in the rainfall-runoff transformation and for difficulties in the runoff ratio estimation, has been estimated. This shows a seasonal dependence and, generally, a dependence on the initial state, prior to an event.

Keywords: antecedent conditions; soil moisture; base flow; runoff coefficient

#### 1. INTRODUCTION

The proportion of total rainfall that becomes runoff during a storm event, represents the runoff coefficient. In the classical 'rational method' it is considered to be a constant, depending on characteristics of the drainage basin, such as surface cover (eg. Dooge, 1957). Some authors proposed a dependence of runoff ratio on the percentage of impermeable catchment area (eg. Schaake et al., 1967; Boughton, 1987). Hebson and Wood (1982), in their study assumed a constant runoff coefficient, interpreted as the percentage of contributing area to runoff generation. The runoff ratio variability is also well documented in the literature, even thought there is no clear conclusion about what factors govern this variability (eg. Gottschalk et al., 1998; Wainwright et al., 2002). The rainfallrunoff transformation is a non linear process. The most important cause of non linearity is represented by the effect of antecedent conditions, consequently the runoff coefficient depends on the initial conditions. It is well known that soil moisture is a major control on catchment response: antecedent soil moisture conditions have been used to represent variability of the CN in the SCS method for predicting direct runoff (eg. Rallison and Miller, 1981). But soil moisture measurements are usually not widely available and a lot of point measurements are needed to get

either an average or an antecedent catchment state. Antecedent soil moisture conditions can be simulated using models (either lumped or distributed) provided standard climate data are available. Alternatively, some other easily measured hydrological variables that are even if only approximately related to soil moisture conditions, may be considered. In this study, we assess the role of antecedent conditions, using directly observed soil moisture and base flow (derived from daily flow records), as predictors of catchment state.

The dependence of runoff ratio on antecedent conditions can also be assessed by means of a conceptual framework. We expect catchment state to be variable from event to event and we might expect seasonal variability to be particularly important in many climates. In this paper we show that the parameter of a conceptually based stochastic model for stream flow simulation, which considers the watershed response as deriving from the response of two linear reservoirs in parallel, is found to be seasonal dependent. The result in each section will be discussed distinguishing between different climatic regions.

#### 2. PREDICTORS OF CATCHMENT STATE

As previously stated, soil moisture measurement, as an indicator of antecedent conditions prior to events, is not as accessible as rainfall or stream flow measurement. For practical purposes, different indexes have been defined which can represent, even if approximately, soil moisture condition and which can be more easily measured. Proposed indexes include base flow at the beginning of the storm, basin evaporation, (since this phenomena is related to soil moisture depletion) and antecedent precipitation index API (Chow, 1964). The latter has received the widest use because precipitation is readily measured.

There are several reasons why base flow is conceptually attractive for predicting catchment state. Many moisture storage based models that simulate saturation excess runoff (eg. TOPMODEL, Beven and Kirkby, 1979; Boughton, 1987) predict the runoff to be proportional to the saturated contributing area which is related to the simulated soil water content and subsurface flow. More confirmation comes from some field studies involving isotope and chemical analysis (eg. Rice and Hornberger, 1998) showing that the pre-event water can represent a substantial component of the stream flow. Moreover base flow time series are more widely available than soil moisture time series, since they can be derived from stream flow measurements analysis and are therefore potentially useful in stream flow forecasting models.

In order to investigate base flow and soil moisture properties as predictors of antecedent conditions prior to rainfall events, an event based rainfallrunoff analysis was undertaken for an extensive database including a wide range of catchment sizes, climate and basin features. The major step in this analysis was the estimation of the runoff coefficient, on an event basis, as the ratio of the quick flow volume and the rainfall volume during the event itself. This was done using a digital filter hydrograph separation analysis (Nathan and McMahon, 1990). We then examine the relationships between the runoff ratio, pre-event base flow and pre-event soil moisture through correlation and regression analysis, distinguishing between different climatic regions.

# 2.1. The database

The database consists of about 380 catchments, with drainage areas ranging between 0.3 and 2000  $\rm km^2$ , across different countries (Australia, New Zealand and Italy). Rainfall and stream flow are available at different time resolution (daily and

hourly) for different basins. Land cover properties and soil properties are also available. Some of the catchments are the object of an intensive field hydrology study, known as MARVEX (MAuranghi River Variability EXperiment) (Woods et al. 2001). In these catchments, detailed soil moisture data via permanently installed CS615 water content reflectometers was available. For our purpose, simulated soil moisture data were derived from the SIMHYD rainfall-runoff model (Chiew et al., 2002) for the remaining catchments.

It has to be stressed here that since for most of the analysed catchments, the soil moisture data are simulated data, the analysis results, in terms of soil moisture dependence, may be in part affected by model simulation. However, since the analysis results of simulated data were similar to the analysis result of measured data, we believe that the investigations are still useful. Possibly indicative wetness might be all that is needed to establish antecedent conditions prior to an event, rather than complex water balance models or detailed spatial measurements. Gravson et al. (1998) suggested that a network of a limited number of moisture sensors at automatic weather stations could provide reliable estimates of areal mean soil moisture time series that could potentially be used for antecedent condition data.

# 2.2. Data analysis results

Examining the mean and the coefficient of variation of the runoff ratio  $(C_f)$ , it is possible to infer that climate and catchment area represent the main factors dominating the proportion of rainfall that becomes runoff. Large catchments showed more uniform behavior (low coefficient of variation values), as their properties are an averaged value, while the hydrological response of small catchments tends to be more variable, but quantitative scale effects do not appear to be obvious, as we will discuss later. In this study we will focus on the climate dependence. In arid areas the coefficient of variation is greater than in humid areas, where the dynamics of the saturated contributing area is less. The introduction of a wetness ratio I (eg. Berger et al., 2001), representing the climatic forcing, computed as the ratio between the mean annual precipitation P and the mean annual potential evaporation ETP, enables grouping of a large number of basins in three main climatic regions: a very humid environment with I > 1.5, a humid environment with 1.5 > I > 0.5 and an arid environment with I < 0.5. Summary results will be discussed for each region.

The results of correlation analysis indicate that both pre-event base flow and pre-event soil moisture are strongly related to the runoff coefficient (correlation coefficient ranging between 0.60 and 0.94) in humid environments, whereas in arid and very humid areas, the correlation is weak (< 0.40), being weakest for very humid environments. If C<sub>f</sub> does depend on pre-event conditions, even though the relationship is not strong, we can introduce the concept of conditional probability, assessing how the probability of a given value of C<sub>f</sub> is dependent on a given value of antecedent condition that has occurred. The conditional probability that the runoff ratio is greater than the average runoff ratio, given that the pre-event soil moisture (preevent base flow) is greater than the average preevent soil moisture (pre-event base flow) are listed in Table 1.

**Table 1.** Conditional probability of  $C_f$  to preevent soil moisture and to pre-event base flow. Grey cells indicate no significant information.

| Climatic region | Pre-event soil<br>moisture | Pre-event base flow |  |
|-----------------|----------------------------|---------------------|--|
| Arid            | 50%                        | 50%                 |  |
| Humid           | 56%                        | 85%                 |  |
| Very humid      | 83%                        | 88%                 |  |

To test their ability in runoff prediction, a simple empirical model equation,  $C_f = \alpha B f^{\eta}$ , has been fitted to the observed runoff coefficients, where  $C_f$  is the runoff coefficient, Bf is the pre-event base flow (or pre-event soil moisture) and  $\alpha$  and  $\eta$  the regression coefficient and the regression exponent respectively. The  $\eta$  coefficient is considered a measure of the system linearity.

**Table 2.** Base flow performance at predictingrunoff generation over the whole dataset (averagevalue).

| Climatic region | SE    | Е     | r <sup>2</sup> |
|-----------------|-------|-------|----------------|
| Arid            | 0.063 | 0.009 | 0.317          |
| Humid           | 0.113 | 0.338 | 0.671          |
| Very humid      | 0.133 | 0.089 | 0.170          |

**Table 3.** Soil moisture performance at predicting runoff generation over the whole dataset (average value).

| Climatic region | SE    | Е     | $r^2$ |
|-----------------|-------|-------|-------|
| Arid            | 0.059 | 0.209 | 0.374 |
| Humid           | 0.112 | 0.354 | 0.673 |
| Very humid      | 0.129 | 0.143 | 0.271 |

Base flow and soil moisture performance have been compared by means of simple standard indicators as the coefficient of determination, the standard error and the coefficient of efficiency (Tables 2 and 3) as average value over the whole dataset. Overall, we observed that r<sup>2</sup> assumes high to moderate values in humid environment for both soil moisture and base flow, whereas the goodness of fit is very poor in arid and very humid areas (although soil moisture is a little better than base flow). In very humid areas, the coefficient of variation of base flow is low, thus the range of variation of one of the variables involved in the regression analysis is quite narrow and, consequently, the regression results are affected. But the reason for a weak correlation may possibly be also a physical one. Using the base flow as a predictor of catchment state assumes the runoff generation by the saturation excess mechanism. In humid areas, with moderate values of rainfall intensity, surface runoff generation from saturated areas is more likely to occur. In arid areas the base flow coefficient of variation is very high and the goodness of fit is poor because of many events that occur with dry antecedent conditions, possibly generated by the infiltration excess mechanism. Analysing the n regression exponent, over all investigated catchments, assumed as a measure of system linearity, it is also apparent (although not shown here) that the relationship between runoff ratio and pre-event base flow is more linear than the relationship between runoff ratio and pre-event soil moisture and that the degree of linearity is higher in humid areas. As stressed before, scale effects do not appear to be obvious and climate dependency seems to dominate scale dependency. There is no simple linear relationship between area and the coefficient drainage of determination, which tends to be higher for catchments ranging between 50 and 1000 km<sup>2</sup> but is lower for both smaller and larger catchments, For larger drainage areas, up to 2000 km<sup>2</sup>, r<sup>2</sup> assumes low values possibly because climatic conditions are spatially variable at such large scales.

In Figures 1 and 2, we show the results of the non linear regression analysis for a  $180 \text{ km}^2$  catchment in a humid environment, with a wetness ratio equal to 0.75. Based on a rainfall threshold, 16 events were selected and for each of them the runoff coefficient was estimated and plotted versus the base flow at the beginning of each event. In order to compare results from many catchments, with very different characteristics and very different value in base flow rate, base flow time series were divided by the respective mean value. The coefficient of

determination is 0.88 and 0.85 respectively for the runoff ratio versus base flow relationship and for the runoff ratio versus measured soil moisture relationship. For this basin, we can infer that the simple empirical model,  $Cf = \alpha B f^{\eta}$ , is able to reasonably reproduce, on an event basis, the runoff coefficient values given the antecedent condition values, except for a few large events.



Figure 1. Runoff ratio versus pre-event base flow for a humid catchment (I = 0.75).



Figure 2. Runoff ratio versus pre-event soil moisture for a humid catchment (I = 0.75).

To further investigate the largest events, the residuals of the non linear regression analysis as applied to all 380 catchments, were analyzed as a function of the maximum value of rainfall intensity during the event, and pre-event soil moisture content. It was found that the larger errors are associated with events characterized by highest values of rainfall intensity and/or highest values of pre-event soil moisture, within different seasons. Large events may be generated by infiltration excess runoff, for which base flow and soil moisture are not indicators.

Pre-event base flow and pre-event soil moisture performance at predicting antecedent conditions and thus runoff generation, were also compared with the Bureau of Meteorology API approach, used in the Australian flood forecasting system. We found (although not shown here) that base flow and soil moisture, which represent the integration of the net climatic forcing, are better predictors of runoff generation than API, with important implications in flood forecasting modeling. The seasonal variation in PET is likely to make antecedent rainfall a poor predictor of catchment antecedent state. The API index was found to be a better predictor for only a few catchments in arid environments.

# 3. PROPOSED CONCEPTUAL APPROACH

In the previous section we highlighted how, in different climatic regions, the catchment response is affected by the role of antecedent conditions by means of simple statistical analysis. We now investigate the variability of the catchment response by applying a stochastic (conceptual based) framework, focusing on the seasonal variability and, within the latter, on the event to event variability of the runoff generation. This model leads also to an alternative methodology of hydrograph separation.

The catchment response is considered as deriving from the response of linear reservoirs in parallel, representing contributions to the stream flow from different components, characterized by different response time (Claps and Murrone, 1993). For our purpose we identify the system as made up of only two linear reservoirs, thus stream flow becomes the sum of two components, a fast and a slow component, which are the results of processes. different dominant The slow contribution to stream flow is mainly represented by base flow and deep subsurface flow, whereas the fast contribution is mainly represented by shallow subsurface flow and surface flow. We identify the former and the latter component simply as the base flow and quick flow respectively. The hydrograph is split between the two components, with different response times, according to a parameter  $\beta$ . The slow component is  $\beta$  proportional to the system input, whereas the fast component is  $(1-\beta)$  proportional to the system input. The catchment response u(t), at the time t, can be described as in the following equation:

$$\mathbf{u}(t) = \beta \frac{1}{K_{\rm s}} \exp\left(-\frac{t}{K_{\rm s}}\right) + (1-\beta) \frac{1}{K_{\rm F}} \exp\left(-\frac{t}{K_{\rm F}}\right) \qquad (1)$$

where Ks and Kf are respectively the response time of the slow and fast component.  $\beta$  also represents the proportion the total rain split between the two linear reservoirs.

In the literature many similar models have been proposed, but  $\beta$  has generally been estimated as a constant over all the year, while we know that, physically, it should not be constant. It is strongly dependent on the initial conditions prior to an event, and on many other variables, such as the

infiltration capacity (variable with wetness conditions), vegetation cover (variable over the year), and rainfall characteristics. It is responsible for the non linearity in the rainfall-runoff relationship and it should be at least expressed as a seasonally dependent variable. A first attempt has been recently made by Xu et al. (2002), assigning a monthly variability to  $\beta$  parameter ( $\beta$ variable over the range 0.65-0.75).

Following this direction, we apply the conceptual framework to a number of catchments, estimating  $\beta$  both on an annual and on a seasonal scale. Averaged  $\beta$  values over the whole dataset are listed in Tables 4, 5 and 6. On average the contribution of the slow component to stream flow production was found to be much higher in arid areas than in humid and very humid areas. Few catchments showed a rather constant behavior over the year. It is also apparent that the fast component becomes dominant during wet seasons, as more frequently soil gets saturated allowing higher runoff production from rainfall falling into saturated areas. Moreover the variability of the catchment response, expressed as variance of  $\beta$ , appears to be smaller in very humid basins than in arid basins.

**Table 4.** Mean and variance of annual  $\beta$  over the whole dataset.

| Climatic region   | Mean        |             | Variance     |             |
|---|-------------|-------------|--------------|-------------|
| arid  | 0.48        |             | 0.021        |             |
| humid   | 0.30        |             | 0.016        |             |
| very humid  | 0.17        |             | 0.001        |             |
| Table 5.Meandataset.  | of sea      | sonal β     | over the     | whole       |
| Climatic region   | Jan-<br>Mar | Apr-<br>Jun | Jul-<br>Sept | Oct-<br>Dec |
| Arid  | 0.77        | 0.46        | 0.41         | 0.50        |
| humid   | 0.37        | 0.41        | 0.28         | 0.34        |
| very humid  | 0.21        | 0.20        | 0.15         | 0.17        |
| <b>Table 6.</b> Variance of seasonal $\beta$ over the whole dataset |             |             |              |             |

| Climatic region | Jan-<br>Mar | Apr-<br>Jun | Jul-<br>Sept | Oct-<br>Dec |
|-----------------|-------------|-------------|--------------|-------------|
| arid            | 0.025       | 0.066       | 0.033        | 0.030       |
| humid           | 0.043       | 0.020       | 0.012        | 0.022       |
| very humid      | 0.006       | 0.001       | 0.001        | 0.003       |

Figure 3 shows the results for the Waterfall Left catchment, New Zealand, with 2.6 km<sup>2</sup> drainage

area and a wetness ratio I equal to 1.13. At first we ran the model for the whole time series, estimating a constant value for  $\beta$  equal to 0.67. We then ran the model on a seasonal scale, disaggregating the time series in four time series, one for each season, and estimating  $\beta$  for each series, observing that it is variable over a quite considerable range (0.57-0.81).



Figure 3. Seasonal variability of  $\beta$  (solid line) and average annual  $\beta$  (dashed line).

Figure 3 shows that in this catchment quick flow is mainly contributing to stream flow during wet season (lower  $\beta$  value), whereas slow flow is mainly contributing during dry season (higher  $\beta$  value), according to overall results.

#### 4. CONCLUSIONS

In this paper we assess the role of antecedent conditions on the variability of the runoff ratio, that is on the variability of the catchment response. Two predictors of catchment state, soil moisture and base flow, have been compared in their ability as predictors of antecedent conditions and in their ability in runoff generation prediction, finding comparable results. Particularly, it was found that base flow and soil moisture are well correlated to runoff ratio from humid catchments. Their performance was also compared with an API approach (although only mentioned in this paper), finding that the API index is a poorer predictor except for a few catchments in arid environment. More insights come from a validation of the relationships between runoff ratio and indicators of catchment state, which describe a state curve equation, by means of a conceptual model parameter, looking for a link between the physical processes and the empirical processes interpretation. The  $\beta$  parameter is shown to vary seasonally over the climatic regions we identify in this study. We believe antecedent conditions to be a key point in runoff production and the result of this study will find further application in stream flow generation modeling, which could be improved by the use of base flow or soil moisture predictors of antecedent conditions. In a conceptual stochastic model the non-linearity in the rainfall-runoff relationship will be accounted for by using an IUH dependent on catchment state, described in terms of either soil moisture or base flow. The conceptual approach, with model parameters having conceptual meanings, will enable extrapolation of results to ungauged basins.

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