Fractal Behaviour of Soil Physical and Hydraulic Properties

N. Gupta[a] , R. Rudra[a], G. Parkin[b] and H.R. Whiteley[a]

[a]School of Engineering, [b]Land Resource Science,

University of Guelph, Ontario, N1G 2W1, Canada

Abstract: Physical and hydraulic properties of soils show spatial and temporal variability at different scales. The measurement and understanding of soil properties is essential to describe dominant hydrologic processes/parameters at various scales. Due to self - similar (or scale invariant) properties of fractals and their representation by a single parameter, fractal dimension, they have potential as a descriptive tool for scaling up various parameters. The objectives of this study was to illustrate fractal depiction of spatial behaviour of soil physical and hydraulic properties on a field scale, and to describe their anisotropic feature and possible impact on fractal dimension. The variogram plotted on a log – log scale was used to estimate fractal dimension of data collected from soil core samples and in-situ field measurements. The results indicate that soil properties show fractal behaviour and cannot capture anisotropic variability of soil properties on a field scale. There is a further need to understand the physical interpretation of the fractal dimension and to develop relationships between fractal dimensions and hydrology parameters.

Keywords: *Fractal; Soil Properties; Scaling; Spatial Variability; Fractal Dimension*

1. INTRODUCTION

The spatial and temporal variability of soil physical and hydraulic properties have been widely recognized and investigated for many years. Researchers have examined variability of soil physical and hydraulic properties over small and large areas. Fractal geometry has also become a useful tool in quantifying spatial variability of aggregate mass, particle mass, soil particle surface, surface roughness and hydraulic conductivity (Gimenez et al., 1997; Baveye et al., 1998). Many researchers including Rieu et al. (1991); Filgueira et al. (1999), Liu and Molz (1996); Perfect et al. (1996);Gimenez et al. (1997); Federico and Neuman (1997) and Oleschko et al. (2000) had tried to develop relationships between fractal dimension and soil physical and hydraulic properties. Bloschl & Sivapalan (1995) reported that the concept of fractals can be applied to describe the spatial and temporal variability of rainfall and soil parameters. Fractal relationships may be have some application to extrapolate spatial and temporal variability from one scale to another, yet no technique is available to do so. Therefore, the overall objective of this study has been to explore fractal behaviour of soil physical and hydraulic properties on a field scale. The specific objectives are to examine the fractal behaviour to describe their spatial and temporal variability, and to explore the characteristics of fractal dimension along and perpendicular to the direction of slope.

2. STUDY AREA & DATA COLLECTION

The field experimental study was conducted during 1999-2000 at the University of Guelph campus in Southern Ontario, Canada. The 0.58 ha field had a gently sloping topography with an average slope of about 6 percent. The dominant soil in the study area is Guelph series and belongs to the Grey - Brown Luvisol great soil groups with loam till as soil materials and have good internal drainage. The texture of soil in the field was sandy loam. The field was under grass for the last ten years. The effect of vegetation was not included in this study; therefore, the existing grass cover and its effect on the land surface were minimized by application of an herbicide and conventional tillage using mould - board - plough operations. The experimental field was demarcated into ninety-one square block, each 64 m^2 in area by a 30 cm long wooden stake, inserted 10 cm into the soil at the nodes (Fig. 1). The soil physical and hydraulic properties were determined from collected soil samples and in-situ field measurements at all ninety-one grids. The measurements included dry bulk density, total porosity, particle size distribution, field-saturated hydraulic conductivity (K_{fs}) and suction at wetting front (ψ_f). The ψ_f were determined from soil water characteristics developed by laboratory experiments on the samples collected in the field. The K_{fs} was estimated from infiltration data obtained from Double Ring Infiltrometer (*DRI*) by using the two Term Philip Equation.

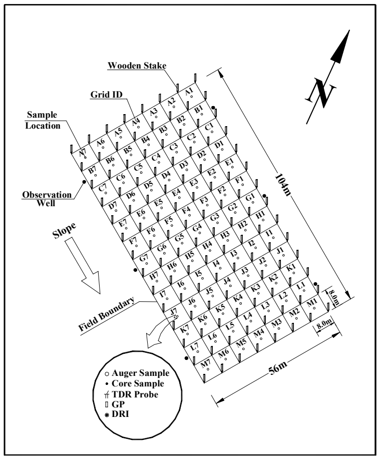

Figure 1. Experimental set-up in the field

3. MATERIALS AND METHODS

The variogram method for estimating fractal dimension (D_f) was based on the assumption that the selected soil property has statistical properties similar to those of fractional Brownian surfaces. The self-similar properties of fractional Brownian motion (fBm) model can be expressed by a power - law variogram in the spatial domain as:

$$\gamma(h) = \frac{1}{2} E[\{Z(x) - Z(x+h)\}^2] = ch^a \quad (1)$$

Where

γ (h) = semi-variance at lag h
h = distance (lag)
x = position in one dimensional space
c = constant of proportionality
a = slope

For a fractional Brownian motion model, a belongs to the interval $0 < a < 2$ and is related to the Hurst exponent, H and subsequently to D_f by the following relationship given by Huang and Bradford (1992):

$$a = 2H \quad (2)$$
$$D_f = E_d + 1 - H \quad (3)$$

where

E_d = Euclidean dimension (Voss, 1985)
H = Hurst coefficient with a range $0 < H < 1$

Taking logarithms on both sides of Equation (1) and by plotting it on the arithmetic graph paper, the semi - variogram approximates a straight line. The fractal dimension (D_f) was estimated from the slope of the fitted line.

4. SEMI VARIOGRAM AND FRACTAL ANALYSIS

A geostatistical software (GS+) for Windows 98, Demonstration Version 3.11 (Gamma Design Software, Michigan, USA) was used to analyse the spatial structure for bulk density, total porosity, soil texture, field saturated hydraulic conductivity, and suction at the wetting front. Isotropic and anisotropic empirical semi – variograms in the east – west (along the slope) and north – south (perpendicular to the slope) directions were calculated. The empirical semi – variogram for bulk density and soil texture were calculated and are shown in Figures 2 and 3, respectively. The anisotropic variogram were plotted in the east - west and north - south direction. The semi - variogram developed along the slope is positioned above semi-variogram perpendicular to direction of slope. Such a trend indicates that spatial variability of bulk density observations is associated with direction.

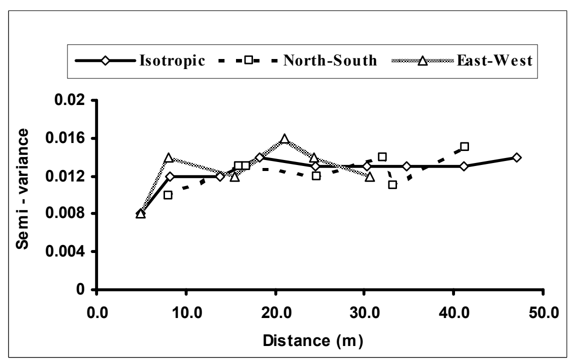

Figure 2. Empirical variogram for bulk density during fall, 1999

The high value of the nugget for the empirical semi-variogram of bulk density indicates that bulk density observations are highly spatially variable within a sample volume. The overall shape of the variogram is representative of transition models (Isaaks and Srivastava, 1989) indicating that values within each data set are highly correlated. The empirical semi - variogram for bulk density shows a spatial structure that can be well represented by an exponential model.

Due to a very small difference observed between the directional and omni - directional (isotropic) semi-variograms for bulk density, the isotropic semi-variograms were used for estimation of fractal dimension. As the total porosity was calculated from bulk density, trends were similar to those of bulk density for isotropic and anisotropic semi – variograms. Therefore, semi-variograms of total porosity have not been shown. The empirical semi - variograms for soil textural components such as coarse sand, silt, and clay were calculated and are shown in Figure 3. The semi - variogram calculated along the direction of slope is positioned above the semi - variogram perpendicular to the direction of slope for clay. Also, anisotropic semi - variogram for coarse sand and silt along the direction of slope is positioned slightly above the semi - variogram perpendicular to the direction of slope. Such a trend indicates that spatial variability of sand, silt, and clay is also associated with direction.

The observed value of differences in variance of directional semi - variogram for silt indicates that silt particles are more spatial variable than coarse sand and clay particles. These results also indicate that silt particles are more susceptible to detachment, transport, and deposition during rainfall – runoff event. The higher value of nugget for silt indicates that spatial variations of silt particles in a soil sample volume are more than spatial variations for other soil particles. As these data indicate a very small difference between the directional and omni - directional (isotropic) semi - variograms for coarse - sand, silt and clay. Therefore, isotropic semi - variograms were used for estimation of fractal dimension for coarse - sand, silt, and clay.

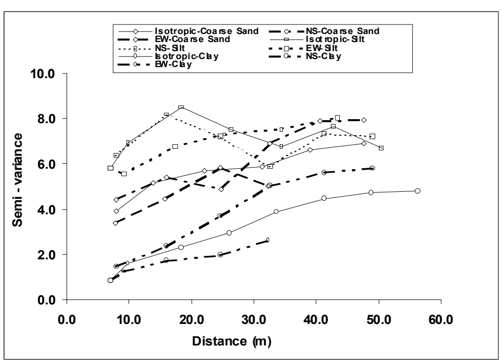

Figure 3. Empirical variogram for coarse sand, silt and clay

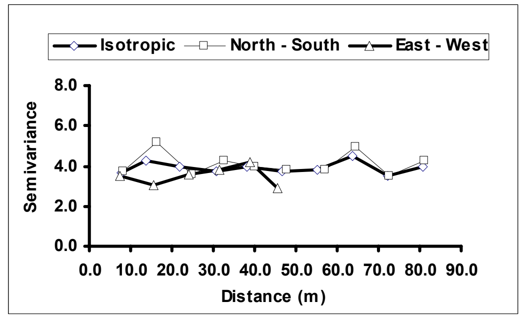

Figure 4. Empirical variogram for K_{fs}

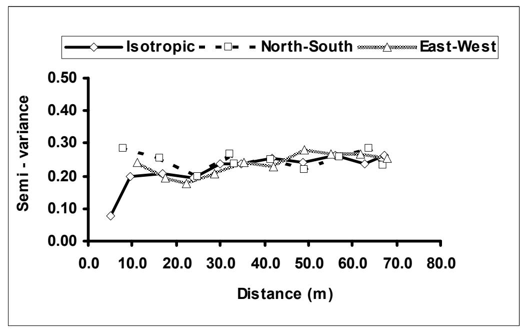

Figure 5. Empirical variogram for suction at the wetting front

The empirical semi - variograms for field saturated hydraulic conductivity and suction at the wetting front were calculated and are shown in Figure 4 and 5 respectively. The semi - variogram calculated along the slope is positioned above semi – variogram perpendicular to the direction of slope. Such behaviour indicates that spatial variability of K_{fs} observations is associated with direction. A high value of nugget for empirical semi- variogram for K_{fs} indicates that K_{fs} exhibits very highly spatially variability. The empirical semi - variograms of K_{fs} also show a poor spatial structure indicating that K_{fs} measurements are not correlated to each other. As the directional and omni - directional (isotropic) semi – variograms of K_{fs} shows a similar pattern, therefore, isotropic semi - variograms were used for estimation of fractal dimension for K_{fs}. The positioning of isotropic and anisotropic empirical semi – variograms for suction at the wetting front (P_f) along the direction of slope is above the semi - variogram perpendicular to the direction of slope indicates that spatial variability of P_f observations is also associated with direction. Also, a low value of nugget suggests that P_f is less spatially variable than saturated hydraulic conductivity. The isotropic semi - variograms for P_f also show a good spatial structure indicating that P_f measurements are correlated to each other and can be represented by an exponential model. Due to very small difference observed between the directional and isotropic empirical – variograms, isotropic semi - variograms were used for estimation of fractal dimension for P_f.

A regression analysis was performed between the logarithms of *((h)* and *h* using the isotropic variogram values for bulk density, total porosity, soil texture, field saturated hydraulic conductivity, and suction at wetting front. The parameters of the straight line fitted to each isotropic variogram for soil physical and hydraulic properties were calculated and are shown in Figures 6 to 9.

4.1 Fractal Dimension

To calculate fractal dimension, a regression analysis was performed between the logarithms of *((h)* and *h.* The plots of logarithms of omni - directional semi - variogram and distance (lag = *h*) for bulk density, total porosity, soil texture, field saturated hydraulic conductivity, and suction at the wetting front are shown in Figures 6 to 9. The least square procedure was used to fit a linear function to the isotropic semi - variogram for all soil properties. The semi - variance values at distances less than the sampling distance were omitted sequentially to obtain the highest value of the coefficient of regression for isotropic variogram. The fractal dimension was computed using equations (2) and (3) from the slope of linear regression given in Table 1. The fractal dimensions for soil properties vary from 2.51 to 2.91 and are in accordance with observations of Wheatcraft and Tyler (1988). These results indicate soil properties exhibit fractal behaviour. As isotropic variograms were used to estimate fractal dimension, therefore, fractal dimension remain invariant with respect to direction on a field scale. Other researchers (e.g. Neuman, 1990; Molz and Boman, 1993) observed fractal behaviour of hydraulic conductivity and concluded that the fractal dimension could be used for scaling over large length scales in a broad variety of geologic media under diverse conditions of flow and transport. The results of this study support the contention that the scale invariant property of fractals can be applied as a scaling rule to represent spatial variability of soil properties over large areas.

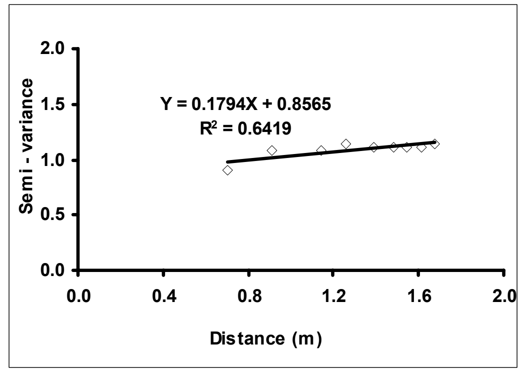

Figure 6. Logarithmic plot of empirical variogram for bulk density

5. CONCLUSIONS AND RECOMMENDATIONS

The fractal geometry provides a statistical tool for characterizing the spatial variability of soil physical and hydraulic properties. The fractal dimension provides a single parameter to describe the spatial variability of soil physical and hydraulic properties. Fractal behaviour can be used to transfer the information across scales by extrapolating properties observed at one scale to properties at other scale. Further research is needed to understand the physical interpretation of fractal dimensions and their application in hydrologic modelling.

Table 1. Fractal dimension of soil properties on a field scale

Parameter	Fractal Dimension (m)
Bulk Density	2.91
Porosity	2.91
Kfs	2.90
Suction at the wetting front	2.82
Coarse sand	2.85
Silt	2.95
Clay	2.57

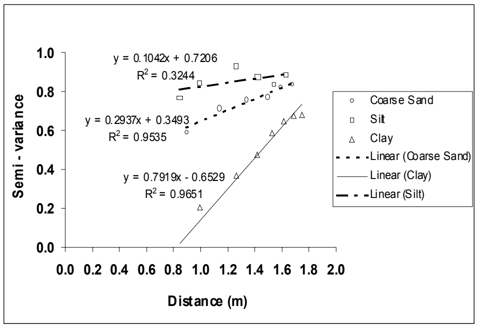

Figure 7. Logarithmic plot of empirical variogram for coarse sand, silt and clay

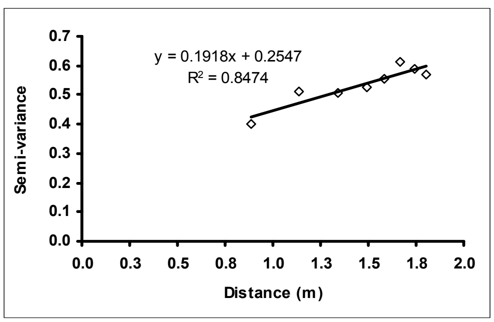

Figure 8. Logarithmic plot of empirical variogram for K_{fs}

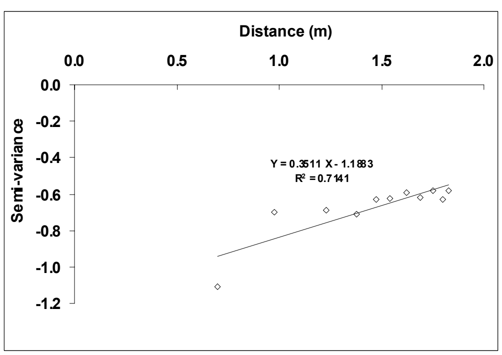

Figure 9. Logarithmic plot of empirical variogram for suction at the wetting front

6. REFERENCES

Baveye, P., J.Y. Parlange, and B. A. Stewart, Fractals in Soil Science. *Advances in Soil Science*, CRC Press, Boca Raton, FL, 377 pp. 1998.

Bloschl, G. and M. Sivapalan, Scale issues in hydrological modelling : A review. In : *Scale issues in hydrological modelling,* Advances in Hydrological Processes : by J. D. Kalma and M. Sivapalan. 1995.

Federico, D.V. and S. P. Neuman, Scaling of random fields by means of truncated power variograms and associated spectra. *Water Resources Research*, 35(5),1075- 1085, 1997.

Filgueira, R. R., L. L. Fournier, G. O. Sarli, A. Aragon and W. J. Rawls Sensitivity of fractal parameters of soil aggregates to different management practices in a Phaeozem in central Argentina. *Soil & Tillage Research*, 52 - 217, 1999.

Gimenez, D., R. R. Allmaras, D. R. Huggins, and E. A. Nater, Prediction of the saturated hydraulic conductivity - porosity dependence using fractals. *Soil Science Society American Journal*, 61-1285,1997.

Huang, C., and J. M., Bradford, Applications of a laser scanner to quantify soil micro-topography. *Soil Science Society America Proceedings.* 56, 14, 1992.

Isaaks, E.H. and R.M. Srivastava, An introduction to applied geo-statistics. Oxford University Press, New-York. 561 pp.

Liu, H. H. and F. J. Molz, Discrimination of fractional Brownian motion and fractional Gaussian noise structures in permeability and related property distributions with range analysis. *Water Resources Research*, 32, 2601, 1996.

Molz, F. J. and G. K. Boman, A stochastic interpolation scheme in subsurface hydrology, *Water Resources Research*, 29(11), 3769-3744, 1993.

Neuman, S. P. Universal scaling of hydraulic conductivities and dispersivities in geologic media. *Water Resources Research*, 26, 1749 – 1758, 1990.

Perfect, E., N. B. McLaughlin, B.D. Kay, and G. C. Topp, An improved fractal equation for the soil water retention curve. *Water Resources Research*, 32(2), 281- 287, 1996.

Rieu, M., and G. Sposito, Fractal fragmentation, soil porosity, and soil water properties : I. Theory.*Soil Science Society America Journal*, 55, 1231 – 1238, 1991.

Tyler, S. W. and S. W. Wheatcraft, Application of fractal mathematics to soil water retention estimation. *Soil Science Society of America Journal*, 53, 987-996, 1989.

.