Development of a Simple Cascading Bucket Model for Hillslope Hydrology

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Abstract: Modelling is a valuable tool for hydrological study. Models vary in their numerical and parametric complexity, input requirements, possible response variables, and the certainty of the outputs. Model selection therefore depends on the available data for input parameters, the desired outputs, and computational resources. 'HILLS', a physically-based distributed model, was used in previous research into hillslope hydrology in south-east Australia. The study investigated the sensitivity of soil, topography, and rainfall to generating subsurface lateral flow. Model instability and overparameterisation created uncertainty in the results. Consequently for further study into hillslope hydrology in south-east Australia, a simple cascading bucket model was developed, using data from a field monitoring site near Holbrook, New South Wales. The detailed field information enabled model validation on the runoff and subsurface lateral flow from the hillslope outlet, as well as water redistribution within the hillslope, reflected by the soil moisture content and depth of watertables. This paper provides a description of the cascading model.

Keywords: *Bucket model; Hillslope hydrology; Subsurface lateral flow*

1. INTRODUCTION

Modelling tools are commonly used in hydrological studies at hillslope to regional scales. There are three basic types of models used in hydrology (Ye *et al*, 1997) that vary in their numerical and parametric complexity, input requirements, response variables and the certainty of the outputs.

Empirical models are generally a simple, direct and often abstract relationship between an input and output (Hook *et al*, 1998). Conceptual models lump several processes together and link each element with simple mathematical equations (Ye *et al*, 1997). Lumping processes minimizes the number of model parameters but a long-term data set may be required to determine their values. Physically-based distributed models are generally the most complex model type because they explain each process specifically "based on our understanding of the physics of the hydrological processes" (Beven, 1989:405). Generally a large number of parameters are required to explain the complexity of distributed models.

Greater complexity does not mean the model is better, and simple lumped models have performed as well as, or better than more complex alternatives (Jakeman and Hornberger, 1993, Jakeman *et al*, 1994). Overparameterisation and a lack of appropriate data for parameterisation are a particular concern with complex models (Jakeman and Hornberger, 1993, Jakeman *et al*, 1994)

In previous research on hillslope hydrology in south-east Australia (Ticehurst *et al*., 2001), 'HILLS' (Smith and Hebbert, 1993), a physically-based distributed model, was used to investigate the sensitivity subsurface lateral flow (SLF) (or throughflow) to of soil, topography, and rainfall. The 'HILLS' model appeared well equipped for a study on SLF as it has two soil layers, enables the topography to be defined at many points along the hillslope, and accounts for variable rainfall intensity. However it was not able to account for changes in vegetation down the hillslope, and model instability hindered an investigation into the influence of a rising watertable. Instability and uncertainty, was possibly due to overparameterisation.

Consequently a simpler alternative for hillslope modelling was sought. Grayson *et al* (1992) highlighted the importance of using field data when developing models. Therefore field data from a hydrological site near Holbrook, south-east Australia were used to assist in the development of a simple cascading bucket model for hillslope scale research. This paper discusses model development.

2. HILLSLOPE HYDROLOGY

The general processes that drive hillslope hydrology are now well understood. Water enters the system as precipitation. The vegetation cover may intercept some of the rainfall. Some of this water evaporates, while the rest moves through the vegetation, as stem flow or throughfall, to the soil surface. The water then either infiltrates into the soil, or moves laterally over the surface, where it can infiltrate further down the slope. Water that has infiltrated is redistributed throughout the soil profile depending on the soil properties and hillslope gradient. If impeding layers exist in the soil profile, then vertical drainage will be restricted (Lehman and Ahuja, 1985). This may generate saturated conditions, and move a significant volume of water laterally down the slope on top of the impeding layer. This is known as subsurface lateral flow. Water is lost from the hillslope as evapotranspiration, surface runoff, SLF, or deeper groundwater movement called 'base flow'.

3. FIELD MONITORING SITE

The field site is located near Holbrook, south-east Australia (see Ticehurst *et al*., 2001 for location map). It is a small subcatchment (2.96ha), consisting of a single hillslope. The hillslope does not contain significant drainage lines and is quite planar. There is a series of 'steps' down the slope, which appear to be the result of a sequence of colluvial landslips. The gradients range from 21%, on the steeper slopes, to 8% below the break of slope.

The soils at the site are derived from the granite bedrock. On the upper slopes the soil is a Red Chromosol (Isbell, 1996), interspersed with Rudosols on the steeper slopes where there is significant rock outcrop. Below the break of slope the soil is a Yellow Chromosol, which overlies a buried Sodosol. The contact between the current and buried soil is dense and sheered. The buried soil is brown-yellow, sandy clay, which becomes increasingly mottled with depth. Below this is heavy grey clay above bedrock.

In 1993, 0.21ha of what is now the monitoring site was planted with a tree belt of *Eucalyptus Saligna* (Sydney Blue Gum) and *Acacia Melanoxylon* (Blackwood).

3.1. Monitoring at the Field Site

Hydrological response at the field site has been monitored since September 2001. A meteorological station is located at the bottom of the hillslope. A neutron moisture meter (NMM) was used to measure the soil moisture at up to 6 meters depth at 14 locations across the site. 20 piezometers measure the watertable in the A horizon, on the boundary of the current and buried soil, and on the bedrock. A subsurface trough was used to measure the SLF in the A horizon at the catchment outlet. 74 meters of trough was installed, as in (Stevens *et al*., 1999), by digging a narrow trench down to the A/B horizon boundary, lining the bottom and downslope side of the trench with black plastic, running a slotted irrigation pipe along the bottom, then back-filling with the soil that had been excavated. A surface trough was used to measure overland flow.

3.2. Conceptual Hydrological Model

Field data and observation suggest there are three flow paths for lateral water movement down the hillslope. In the first water percolates deep into the profile on the upper slopes then flows laterally on top of the bedrock down the slope. The water accumulates at the bottom of the slope resulting in a permanent watertable. The second is through the A horizon below the break of slope. And the third flow path is surface runoff from infiltration excess or saturation excess flow.

4. MODEL DESCRIPTION

The model developed for this research is a simple two-dimensional cascading bucket model. A conceptual diagram of the model is presented in Figure 1.

The hillslope is divided into horizontal units called *cells*, *j*. Each cell consists of several *layers*, *m*. Layers are divided into '*sublayers*', *h*. Each layer has a saturated and unsaturated water store. Lateral flow occurs on the surface, and in the saturated store of any layer. Evapotranspiration is extracted from both the saturated and unsaturated sublayers, and evaporation occurs directly from a vegetation interception store. All temporal inputs and outputs are given as a value per time-step, which indirectly sets the time-step.

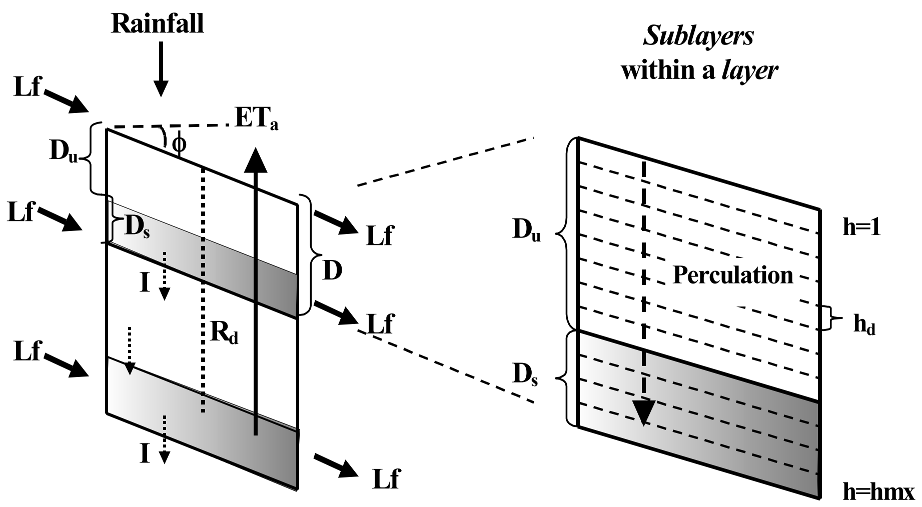

Figure 1. Conceptual representation of the cascading bucket model for a single cell with 2 layers, and 12 sublayers in each

4.1. Inputs

The model requires two data time series to define the rainfall and evapotranspiration and one parameter set defining the physical characteristics of the site. Rainfall and potential evapotranspiration are entered as millimeters of water per time-step. Evapotranspiration data are the standard reference evaporation data, adjusted for the vegetation type, using crop coefficients. Potential evapotranspiration $PET_v(j,t)$ is given for each cell j, at time t, to allow for variation in vegetation type.

A parameter set is used to define the topography, vegetation and soil physical and hydraulic properties. The rooting depth Rd (m), and a maximum vegetation interception store S_{vmx} (mm) is assigned for each cell.

The width of each cell at the downslope boundary w (m), the length L (m), and the surface gradient below the horizontal (Φ in degrees) define hillslope topography.

The soil properties required for each layer are the porosity G (vol/vol) and the depth of the layer at the downslope side of the cell D (m). The hydraulic properties required for each layer are the vertical saturated hydraulic conductivity K_{sat} (mm of water per time-step), field capacity θ_f (vol/vol), wilting point W_p (vol/vol), and the slope of the pore size distribution λ. It is assumed that the horizontal and vertical conductivity are equal within each layer. The initial volumes of the unsaturated S_u and saturated S_s stores are also required.

4.2. Model Description

Water redistribution calculations begin with rainfall interception by vegetation, followed by calculation of the surface runoff and infiltration. Next redistribution of water within the top soil layer and lateral flow are calculated. This is repeated for each layer in the cell at the top of the slope. Evapotranspiration is then extracted from layers within the plant rooting depth. This is repeated for each cell down the hillslope.

Vegetation interception

Vegetation cover is assumed to be uniform and complete, so no rain falls directly onto the soil surface. Rainfall must exceed the maximum vegetation inception store S_{vmx} before any water reaches the surface. Water that is intercepted by the vegetation is lost as evaporation $ET_v(j,t)$, unless limited by the $PET_v(j,t)$. Water that isn't evaporated remains stored till the next time-step. Rainfall reaching the surface is considered effective rainfall $R_{eff}(j,t)$.

Subdivision of each layer

Each layer is divided into '*sublayers*' h purely to control the rate of vertical water movement through the layer. The total number of sublayers hmx is the same in each layer. hmx is set as the maximum number of time-steps required for water to transit vertically through any layer in the hillslope under saturated conditions (i.e. $1000D/K_{sat}$). Each sublayer is either saturated, partially saturated or unsaturated. The store volume h_{vol} and depth h_d of each sublayer varies between layers, because of differences in the depth and porosity of each layer.

$$h_{vol}(m,j) = S_t(m,j)/hmx \tag{1}$$

$$h_d(m,j) = D(m,j)/hmx \tag{2}$$

where $S_t(m,j)$ is the total water store for layer m, cell j.

Vertical percolation of water within a layer

The water storage $Sw(h,m,j,t)$ in sublayer h, of layer m, of cell j, at time t is entered as a fraction of the possible saturated store. Under saturated conditions vertical water movement equals the saturated hydraulic conductivity. In unsaturated conditions water is propagated through the sublayers at a rate equal to the unsaturated hydraulic conductivity (equation 3) (Brooks and Corey, 1964). The unsaturated store is expressed as a fraction of the possible saturation for that depth (unsaturated depth D_u × porosity G). The pore size distribution λ and saturated hydraulic conductivity K_{sat}, give the unsaturated value K_{un} (mm of water per time-step) according to

$$K_{un}(m,j,t) = \left(\frac{S_u(m,j,t)}{D_u(m,j,t) * G(m,j)} \right)^{2/(\lambda+3)} K_{sat}(m,j) \tag{3}$$

The unsaturated conductivity is converted to a depth the water flows in a time-step, given as a number of sublayers h_{ku}.

$$h_{ku}(m,j,t) = K_{un}(m,j,t) / h_{vol}(m,j) \tag{4}$$

Water is unable to move vertically under gravity unless it is greater than field capacity θ_f. Therefore the propagation of water proceeds by moving the store in each sublayer $Sw(h,m,j,t)$ that exceeds field capacity θ_f, down the layer by h_{ku} sublayers. Saturation may occur at the top of a layer following infiltration but the depth of saturation D_s used to calculate lateral flow is from water accumulation at the bottom of a layer.

Movement of water between layers

Infiltration $I(m,j,t)$ is the vertical movement of water into one layer, or sublayer, from the one above. The drainage $D_r(m,j,t)$ is calculated as in APSIM-SoilWat (McCown, *et* al, 1996), with $cos(\phi(m,j))$ added to account for the component of the gravitational acceleration perpendicular to the boundary

$$D_r(m,j,t) = [(Sw(h,m,j,t) - \theta_f(m,j,t)) \\ *swcon(m,j)] * \cos(\phi(m,j)) \tag{5}$$

where $swcon(m,j)$ is a fraction of the water that can drain in a time-step, which reflects the hydraulic conductivity and structure of the layer.

The store volume available to accept more water $A_v(m,j,t)$ can limit infiltration. The available store is the possible store in the sublayers drained in the previous time-step, minus their current water content.

For surface infiltration, the amount available to infiltrate $Sw(h,m,j,t)$ is the sum of the effective rainfall and the lateral flow from the upslope cell.

Lateral flow

Lateral flow $L_f(m,j,t)$ is the volume of water that moves from the saturated zone of one layer to the corresponding layer of the downslope cell. The potential lateral flow $PL_f(m,j,t)$ is a function of the length $L(j)$ and width $w(j)$ of the cell, flow depth $F_d(m,j,t)$, and the hydraulic conductivities of the current layer $K_{sat}(m,j)$ and the corresponding layer in the downslope cell $K_{sat}(m,j+1)$ (equation 6). The flow depth is the depth of saturation $D_s(m,j,t)$, adjusted for topographic gradient by $cos(\phi(m,j))$. The lateral flow is limited by the volume of water in the saturated store available to move laterally $S_s(m,j,t)-\theta_f(m,j)$, and the store available to accept the water in the downslope cell $A_v(m,j+1,t)$. The available store $A_v(m,j+1,t)$ is adjusted for the change in length L and width w between cells according to equation 7, to give the lateral flow (equation 8).

$$PL_f(m,j,t) = \min(K_{sat}(m,j), K_{sat}(m,j+1)) \\ F_d(m,j,t)w(j)/(L(j)w(j)) \tag{6}$$

$$A_v(m,j+1,t) = (S_t(m,j+1) - S_s(m,j+1,t) - S_u(m,j,t)) \\ L(j+1)w(j+1))/(L(j)w(j)) \tag{7}$$

$$L_f(m,j,t) = \min(PL_f(m,j,t), S_s(m,j,t) - \theta_f(m,j), \\ A_v(m,j+1,t)) \tag{8}$$

For surface runoff $L_f(0,j,t)$, the depth of saturation is

$$L_f(0,j,t) = (R_{eff}(m,t) + L_f(m,j-1,t)) \\ \cos(\Phi(j,t)) \tag{9}$$

Evapotranspiration

As a soil dries out it becomes increasingly difficult for vegetation to extract the remaining water from the soil. To account for this, the potential evapotranspiration $PET_v(j,t)$, minus any direct evaporation from the vegetation interception ET_v, is adjusted depending on the soil water store within the rooting depth. The soil water content is expressed as a fraction of

Table 1. Parameter input values for the topography, vegetation and soils.
* estimated from other site measurements,
** estimate from literature.

Parameter	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5
	Modelling Unit				
Surface gradient 'ϕ' (^{o})	12	21	9	8	8
Vegetation type	pasture	pasture	pasture	tree	pasture
Interception Store 'S_{vmx}' (mm)	0.1**	0.1**	0.1**	0.2**	0.1**
Cell length 'L' (m)	56	194	96	51	50
Cell width 'w' (m)	15.6	63.2	66.2	73	73
Rooting depth 'Rd' (m)	1.5*	1.5*	1.5*	3.0*	1.5*
Saturated hydraulic conductivity 'Ksat' (mm/hr)					
layer 1	278*	278	577	573	547
layer 2	1.38*	1.38	0.32	1.94	9.68
layer 3	133*	133*	133*	133*	133
Pore size distribution 'λ'					
layer 1	0.142*	0.142	0.140	0.089	0.193
layer 2	0.071*	0.071*	0.071*	0.071*	0.071
layer 3	0.100*	0.100*	0.100*	0.100*	0.100*
Porosity 'G' (%)					
layer 1	41*	41	39	40	40
layer 2	42*	42	38	38	38
layer 3	40*	40*	40*	40	40*
Layer depth 'D' (m)					
layer 1	0.58*	0.58	0.45	0.45	0.45
layer 2	1.33*	1.33	1.05	1.35	1.95
layer 3	4.15*	4.15	4.5	4.2	4.9
Field capacity 'θf' (fraction of total water store)					
layer 1	0.19*	0.19	0.21	0.22	0.28
layer 2	0.25*	0.25*	0.25*	0.25*	0.30
layer 3	0.25*	0.25*	0.27*	0.27*	0.27*

the possible available water for that depth α. The final potential evapotranspiration $PET(j,t)$ is

$$PET(j,t) = \alpha \times PET_v(j,t) \tag{10}$$

Thus when the soil is completely saturated to the bottom of the rooting depth, $PET(j,t)$ equals $PET_v(j,t)$, and when it is dry beyond the wilting point, $PET(j,t)$ equals zero. The actual evapotranspiration ET_a from each cell is

$$ET_a(j,t) = \alpha \times PET(j,t) \tag{11}$$

The actual evapotranspiration from each sublayer $ET_{ah}(h,m,j,t)$ is found by removing the same fraction of soil water α, from the soil water above the wilting point in each (equation 12). Therefore wetter sublayers lose more water than drier ones.

$$\begin{aligned} ET_{ah}(m,j,t) = \alpha \times \min(PET(j,t)/n), \\ \max(S_w(h,m,j,t) - W_p(m,j),0) \end{aligned} \tag{12}$$

where n is the number of sublayers within the rooting depth.

4.3. Outputs

The model simulates the surface runoff and lateral flow for each layer at each time-step. It also gives the actual evapotranspiration from each cell for each time-step. For model validation the volume in the unsaturated store and the depth of saturation are also available.

4.4. Boundary Conditions and Assumptions

It is assumed that the vertical boundary at the top of the hillslope is on the catchment boundary so no run-on occurs into the top cell. The vertical boundary at the bottom of the hillslope is manually set by the coefficient Lf_b. It is a value between 0 and 1, which controls the flow depth (equation 13). Therefore when this flow depth F_d is used in equation 8, it controls the lateral flow out of the hillslope.

$$F_d(m,j,t) = D_s(m,j,t) \times Lf_b \times \cos(\Phi(m,j)) \tag{13}$$

The horizontal boundary at the bottom of the lowest layer for each cell is considered to be impermeable, so no water flows out of the hillslope as vertical drainage.

It is also assumed that the water in the saturated store sits parallel to the bottom of the layer.

5. METHODS

5.1. Input Data

Temporal field data was measured at 6-minute intervals. Rainfall was directly measured at the field site. Potential pan evaporation was estimated using the field site meteorological station data as input into the Penman Combination Equation calibrated for south-east Australia (Meyer, 1999) and adjusted for vegetation type with crop factor values of 0.85, 1.20 and 2.38 for pasture, lucerne and trees, respectively (Meyer *et al*., 1999).

The hillslope was discretized on the basis of topography, soil properties and the conceptual hydrological model into 5 cells, each with 3 layers. The cells are numbered 1 to 5, starting at the top of the hillslope.

The physical parameters were determined at several locations over the field site. Parameter values not directly measured for a cell or layer were estimated from other site values or literature. Soil cores were collected to determine the vertical hydraulic conductivity, porosity, soil moisture characteristic, and field capacity (at 1m suction) (Table 1). The hydraulic conductivity of deeper soil horizons was measured using a tube well permeameter. The slope of the pore size distribution 'λ' was estimated from soil moisture characteristic. Rooting depth was estimated based from changes in soil moisture with depth measured by the neutron moisture meter, and the vegetation interception store was estimated from literature (Dunin *et al*., 1988). Topography was defined from a detailed digital elevation model of the site.

5.2. Model Calibration and Validation

The model was run using half-hourly time-steps from 5 September 2001 until 29 October 2002. The boundary coefficient Lf_b was manually calibrated using the first half of the data, and second half of the data was used for model validation. Surface runoff and SLF from the 'A' layer (i.e. through the A horizon) were compared to the field data from the hillslope outlet. The soil moisture data from the neutron moisture meter access tubes was used to validate the changes in the unsaturated store S_u and piezometer data were compared with the depth of saturation D_s.

6. FURTHER RESEARCH

This paper presents a detailed description of a simple cascading bucket model for hillslope hydrology. It was developed to eliminate model instability and uncertainty experienced from a more complex model. It was developed using detailed data from a field site near Holbrook, south-east Australia. The field data enabled model validation on internal hillslope water distribution, as well as the hillslope hydrographs at the bottom of the slope. Results of the model performance will be presented at the conference.

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