Estimation of evaporation in rainfall-runoff models

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Abstract: The first part of this paper compares two algorithms relating daily evaporation to potential evaporation and simulated soil water content. The first algorithm assumes that the ratio of actual to potential evaporation is a single-valued function of water content. The second algorithm, which appears to be conceptually superior, determines the actual evaporation as the smaller of the potential evaporation and a rate determined by the soil water content. The comparison is based on evaporation data from a deep weighing lysimeter, and soil water data obtained by a neutron moisture meter, the vegetation being a deep-rooted pasture grass. Although there is considerable scatter in the results, the second algorithm performs slightly better than the first. The second part of the paper examines the hypothesis that runoff is only marginally affected by day-to-day variations in evaporation, and thus average monthly potential evaporation data can be used in place of daily data as a model input. Results from 4 models applied to data for 15 Australian benchmark catchments suggest this hypothesis is correct, and furthermore that for some catchments the best fit of all models to runoff data is obtained with unrealistically high values of potential evaporation.

Keywords: Evaporation; Soil water; Rainfall-runoff models

1. INTRODUCTION

The classical experiments of Denmead and Shaw (1962) demonstrated the reduction in plant transpiration from potential evaporation (PE) to a lower value as soil water was reduced. The early hydrological models, such as the original Boughton (1965) model and the Stanford model (Crawford and Linsley, 1966), extrapolated this to modelling catchment evapotranspiration E with the algorithm

$$E = PE\ f(\theta) \qquad (1)$$

where θ is the simulated water content in the soil water store. This form has been continued in most models developed in the USA and Europe, with the compilation in Singh (1995) showing at least 10 of 12 applicable models using (1).

However, Denmead and Shaw's experiments show clearly that $f(\theta)$ should really be $f(\theta, PE)$. An alternative formulation is based on the concept that actual evaporation E is the lesser of the atmospheric water demand, expressed by PE, and the ability of the soil-root complex to transmit water, E_S, i.e.

$$E = \min(PE, E_S) \qquad (2)$$

It is usually assumed that E_S is a linear function of the excess of θ above the wilting point. This function was used in the Australian Representative Basins model (Chapman, 1970), and has been adopted in more recent models developed in Australia, such as MODHYDROLOG (Chiew et al., 1993) and its offshoot SIMHYD (Chiew et al., 2002), and GSFB (Ye et al., 1997).

The difference between (1) and (2) can be most easily exemplified by considering what happens on a day when the PE is reduced and E is limited by E_S. Algorithm (1) would have E reduced in the same proportion as the reduction in PE, while algorithm (2) would have E unchanged, which seems more plausible. In the next section, an attempt is made to compare the ability of (1) and (2) to simulate the value of E.

2. COMPARISON OF ALGORITHMS

2.1 Data

The comparison requires simultaneous site measurements of actual (E) and potential evaporation (PE), together with soil water content profiles. Such data are available from the North Appalachian Experimental Watershed at Coshocton, Ohio (Chapman and Malone, 2002), where actual evaporation is estimated from observations on deep (2.4 m) weighing lysimeters, with an estimated error when percolation is occurring of about 0.4 mm/d without rainfall and about 0.7 mm/d with rainfall. In this study, data from lysimeter 101D for the period 1987-1989 were used.

Measurements of soil water to a depth of 2.4 m at this site were made approximately monthly. For the first 180 mm these were determined gravimetrically, and for 150 mm below that depth by means of a neutron probe.

Potential evaporation was calculated from daily data at the experimental watershed's climate station, using measurements of maximum and minimum temperature, dew point, wind run and solar radiation. The vapour pressure deficit was calculated from the second formula of Lamoreux (1962), and the remaining calculations from the algorithms in Hydrological Recipes (Grayson et al., 1996).

2.2 Analysis

Use of soil water data for evaporation estimation requires first an estimate of the depth of soil within which there are roots contributing significantly to transpiration. At the lysimeter site, the soil is a well drained Dekalb silt loam, and the vegetation is a deep-rooted permanent pasture species. The soil description (Kelley et al., 1975) indicates that plant roots are common down to 635 mm, with presumably some expending deeper.

Figure 1 shows all the measured water content profiles for the period 1987-1989, while Figure 2 shows the average water content θ in the soil above the depths shown; it is this variable that will be used in the evaporation algorithms. The first figure suggests drying caused by transpiration is effective down to about 1 m, while the second suggests even greater depths. In view of this uncertainty, both algorithms were fitted to data for average water contents above depths ranging from 178 to 1143 mm.

Inspection of the E and PE data showed wide variation in the winter months, when the pasture is dormant and snow may lie on the ground. The evaporation algorithms were therefore fitted only to the data between April and September of each year. Given also some missing climate or lysimeter data, the final data set for analysis consisted of 20 observations of the 3 variables E, PE and θ.

Both algorithms have 2 parameters, and fitting was done by finding the values of these parameters which minimised the root-mean-square (RMS) difference between observed and predicted evaporation. The results (Figure 3) show that (2) generally fits the data better than (1), and the best fit is obtained with the average soil water content to a depth of about 1 m; beyond that there is a sharp decline in the fitting ability of both algorithms.

The minimum RMS error for algorithm 2 is 0.98 mm/d, which may be compared with the measurement error of 0.4 - 0.7 mm/d and a mean daily evaporation of 3.6 mm in the data.

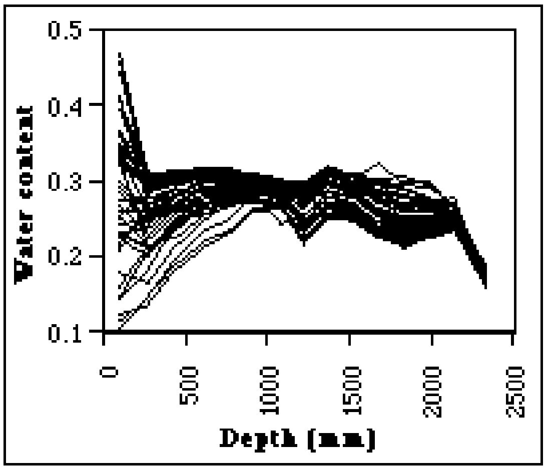

Figure 1. Water content profiles at Coshocton site Y101 for period 1987-89.

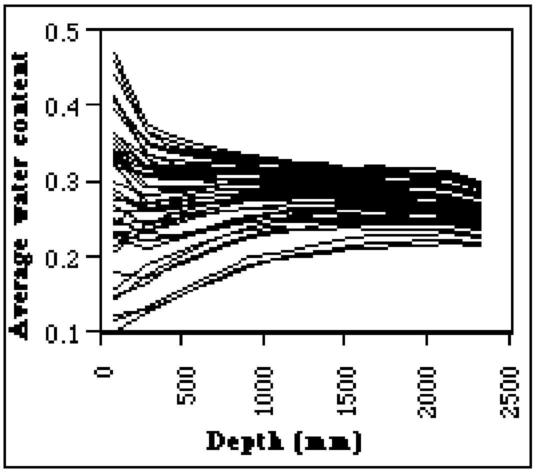

Figure 2. Average water content above the depths shown, site Y101, 1987-89.

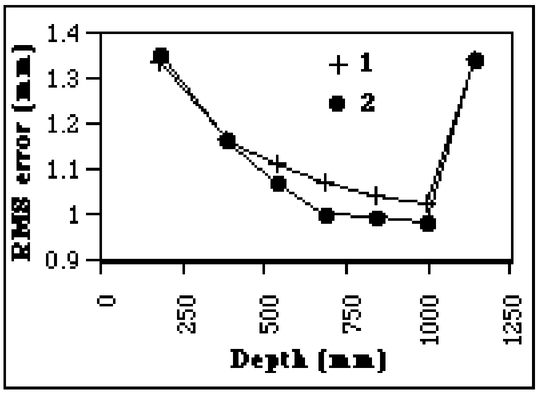

Figure 3. RMS error for (1) and (2) with average soil water measured to the depths shown.

3. MODELLING WITH AVERAGE IN PLACE OF DAILY PE DATA

This part of the paper tests the hypothesis that runoff behaviour at catchment scale is insensitive to daily variations in potential evaporation (PE). The fit of four daily rainfall-runoff models to runoff data has been tested under conditions in which the daily PE data have been replaced by the long-term average PE for each month.

3.1 Description of models

The first 3 models differ only in the way in which modelled rainfall excess is routed to the catchment outlet. Using the usual two-store configuration of the IHACRES model (Jakeman and Hornberger, 1993), the quickflow $x_k^{(q)}$ and slowflow $x_k^{(s)}$ at time step k are added to give the streamflow q_k:

$$q_k = x_k^{(q)} + x_k^{(s)} \qquad (3)$$

The quickflow and slowflow are calculated as linear combinations of the rainfall excess U_k and their values at the previous time step:

$$x_k^{(q)} = \beta_q U_k - \alpha_q x_{k-1}^{(q)} \qquad (4)$$

$$x_k^{(s)} = \beta_s U_k - \alpha_s x_{k-1}^{(s)} \qquad (5)$$

The relative volumes V_q of quickflow and V_s of slowflow are given by

$$V_q = 1 - V_s = \frac{\beta_q}{1 + \alpha_q} = 1 - \frac{\beta_s}{1 + \alpha_s} \qquad (6)$$

so that there are 3 independent parameters in this part of the model.

Model 1 uses the statistical loss module of the 'metric' IHACRES model described by Kokkonen and Jakeman (2001), where the rainfall excess U_k is computed from

$$U_k = s_k^p P_k \qquad (7)$$

where P_k is the measured rainfall at time step k, and p is a parameter which modulates the effect of the catchment wetness index s_k, which in turn is calculated from

$$s_k = c P_k + (1 - \tau_w^{-1}) s_{k-1} \qquad (8)$$

where c is a model parameter. The time constant τ_w is modulated for variations in PE by

$$\tau_{wk} (PE_k) = \tau_w \exp [(c_b - PE_k)/f] \quad (9)$$

where f is a parameter and c_b is a constant at which value $\tau_{wk} (c_b) = \tau_w$.

Model 2 is the 'conceptual' model described by Kokkonen and Jakeman (2001), which envisages a catchment moisture deficit CMD subject to a water balance accounting scheme:

$$CMD_k = CMD_{k-1} - P_k + E_k + U_k \qquad (10)$$

where the rainfall excess U_k is calculated from

$$\begin{aligned} U_k &= c_3 - CMD_k \ , \ CMD_k < 0 \\ &= c_3 \ (1 - CMD_k / c_4) \ , \ 0 < CMD_k < c_4 \\ &= 0 \ , \ CMD_k \geq c_4 \end{aligned} \qquad (11)$$

and the actual evaporation E_k at time step k is defined by

$$E_k = c_1 \ PE_k \exp (- c_2 \ CMD_k) \qquad (12)$$

It will be seen that (12) is of the same form as (1).

Model 3 is a modification of Model 2, in which the algorithm for actual evaporation is of the form of (2):

$$\begin{aligned} E_k &= c_1 \ PE_k \ , \ CMD_k < c_2 \\ &= \min (c_1 \ PE_k \ , c_5 - c_6 \ CMD_k) \ , \ CMD_k \geq c_2 \end{aligned} \qquad (13)$$

Model 4 is an extension of the model SYMHYD (Chiew *et al.*, 2002). Rainfall P_k first enters an interception store of capacity INSC, where it is depleted by evapotranspiration PE_k, leaving an excess EXC_k. Infiltration INF_k is calculated from

$$INF_k = \min [COEFF \exp (-SQ*SMS_k/SMSC), EXC_k] \qquad (14)$$

where SMS_k is the current depth in a soil water store of capacity SMSC, and COEFF and SQ are parameters.

Infiltration excess runoff $SRUN_k$ is obtained as

$$SRUN_k = EXC_k - INF_k \qquad (15)$$

and interflow and saturation excess runoff INT_k from

$$INT_k = SUB * SMS_k/SMSC * INF_k \qquad (16)$$

Recharge to groundwater REC_k is computed by

$$REC_k = CRAK * SMS_k/SMSC * (INF_k - INT_k) \qquad (17)$$

where CRAK is a parameter, and flows into a groundwater store of depth GW_k which produces base flow BAS_k by

$$BAS_k = K * GW_k \qquad (18)$$

K being the daily recession parameter.

Inflow SMF_k to the soil water store is determined by

$$SMF_k = INF_k - INT_k - REC_k \qquad (19)$$

and evaporation E_k from this store by

$$E_k = \min(10 * SMS_k/SMSC \ , \ PE_k) \qquad (20)$$

All stores are adjusted at each time step by the appropriate water balance equation.

SIMHYD is usually run at a daily time step, but calibrated at a monthly time step. To allow for daily calibration, the surface and interflow runoff have here been routed through a linear storage with recession constant KS to give the quickflow as

$$q_k^{(q)} = KS * q_{k-1}^{(q)} + (1-KS) * (SRUN_k + INT_k) \qquad (21)$$

to which the baseflow BAS_k is added to give the total streamflow.

A further extension of SIMHYD has been made by multiplying the values of PE_k by a constant c_1, in line with (12) and (13). This allows for bias in the estimated values of PE.

3.2 Data

The data used in this study were the stream flow records in the data set of Australian catchments prepared by Chiew and McMahon (1993). The locations of the gauging stations are shown in Figure 4. Flows for the 24h period up to midnight were used for the Queensland catchments, and up to 9 am for the other stations. Daily flows for each month. Each model was applied to both the original and the modified data file.

3.3 Model Calibration

Where missing data allowed, each model was calibrated with the first 5 years of data in each file. The first year was taken as a 'run in' period, and fitting was done on the remaining 4 years. Flows in ML were converted to an equivalent depth in mm over each catchment.

For each catchment, the mean daily PE for each month was calculated, and a new data file was prepared, in which the daily PE values in the original file were replaced by the mean daily values for each month.

An extended form of the simplex technique (Nelder and Mead, 1965) was used to minimise the objective function

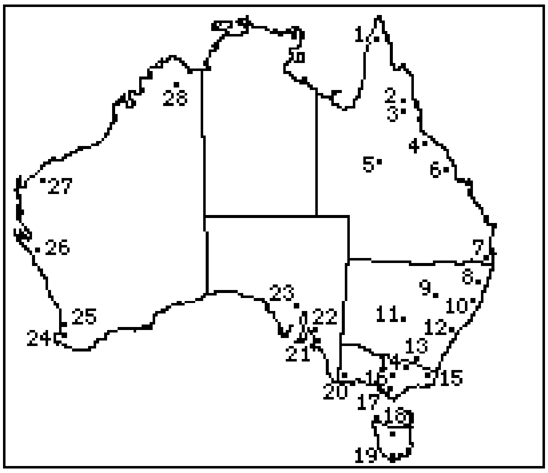

Figure 4. Location of catchments listed in Table 1, from Chiew and McMahon (1993).

$$O = \Sigma (q_k^{0.5} - \hat{q}_k^{0.5})^2/N \qquad (22)$$

where q_k and $\hat{q}_k$ are the measured and modelled daily flows respectively, and N is the number of days modelled.

The performance of each model was measured by three statistics A, B and E (Ye *et al.*, 1997). The mean absolute deviation A is defined by

$$A = \{\Sigma |q_k - \hat{q}_k |\}/N \qquad (23)$$

The bias B is defined by

$$B = \{\Sigma (q_k - \hat{q}_k)\}/N \qquad (24)$$

and the efficiency E by

$$E = 1 - \{\Sigma(q_k - \hat{q}_k)^2 \} / \{\Sigma(q_k - \bar{q})^2\} \qquad (25)$$

where $\bar{q}$ is the mean daily flow.

3.4 Results

Table 1 shows the values of the assessment criterion E for the 15 catchments in which a value of E > 0.6 was obtained by at least one of the models. It is clear that average monthly PE data can be used in place of daily values with no significant impact on the quality of model fitting as measured by this criterion; in fact, for 3 of the models there is a marginal improvement when the monthly data are used.

The purpose of the study was not to compare the relative fitting performance of the models, but it may be noted that each model achieves the highest value of E for at least one data set. The average value of E is higher for Model 3 than for Model 2, suggesting that (2) may perform better than (1); but it must be acknowledged that this may simply

Table 1. Values of model fitting efficiency E with daily (/d) and average monthly (/m) values of PE.

Map Ref.	Catchment	Area (km^2)	Rain (mm)	Model/Dataset							
				1/d	1/m	2/d	2/m	3/d	3/m	4/d	4/m
1	Jardine	2500	1700	0.79	0.79	0.75	0.75	0.66	0.66	0.67	0.61
2	Babinda	39	5400	0.66	0.65	0.66	0.66	0.68	0.69	0.61	0.60
7	Cainable	41	900	0.78	0.79	0.44	0.44	0.60	0.60	0.75	0.78
8	Styx	163	1300	0.56	0.45	0.56	0.68	0.80	0.80	0.57	0.79
10	Allyn	215	1200	0.51	0.51	0.81	0.81	0.80	0.79	0.81	0.75
12	Corang	166	800	0.46	0.54	0.65	0.62	0.67	0.64	0.47	0.58
13	Tooma	114	1700	0.65	0.63	0.56	0.56	0.56	0.55	0.45	0.45
14	Nariel	252	1200	0.80	0.81	0.79	0.76	0.79	0.79	0.75	0.75
16	Dandongadale	182	1300	0.57	0.55	0.68	0.66	0.73	0.77	0.71	0.72
17	Bass	52	1100	0.60	0.62	0.76	0.75	0.73	0.76	0.81	0.80
18	Forth	311	2000	0.69	0.70	0.78	0.77	0.78	0.77	0.77	0.71
19	Davey	686	2100	0.68	0.68	0.74	0.74	0.71	0.70	0.67	0.66
21	Scott	27	950	0.61	0.62	0.72	0.73	0.64	0.68	0.75	0.77
24	Stones	15	1000	0.69	0.63	0.42	0.62	0.66	0.69	0.60	0.62
25	Canning	544	800	0.65	0.68	<0	0.41	0.39	0.44	0.63	0.71
	Average E			0.65	0.64	0.62	0.66	0.68	0.69	0.67	0.69

be due to the 2 extra parameters in Model 3.

Table 2 shows values of the parameter c_1 for Models 2, 3 and 4. This parameter allows for possible bias in the values of PE in the data set, due to the method of estimation. In the Australian benchmark station data set, the PE data were derived from dry and wet bulb temperatures and sunshine hours using Morton's (1983) model. A range of 0.8 to 1.2 for c_1 could be considered reasonable, but Table 2 shows much higher and quite unrealistic values of c_1, for several catchments, particularly those with lower rainfall. These results are generally consistent between models and between the daily and monthly data sets. Values for Model 1 cannot be quoted, as the parameter c_1 is contained within the modulation parameter f in (9).

Values of the bias B were acceptably small (<0.2 mm) for most models and catchments, but most were positive, indicating a slight underestimation of the stream flow. The mean absolute deviation A appeared to correspond roughly with the efficiency E, but values have not been closely analysed at this stage.

4. DISCUSSION

The results described above are surprising, in that they suggest that none of the models tested is correctly simulating the drying processes on a catchment. From the viewpoint of runoff modelling as such, there is certainly a convenience in knowing that average monthly PE data, which can readily be obtained from maps (Bureau of Meteorology, 2001), can be used in place of often tedious calculations from daily climate data, without any deterioration in model performance. However, it appears that there needs to be a re-examination of the concepts on which currently used daily models are based.

Table 2. Values of the PE weighting parameter c_1 , with daily (/d) and monthly (/m) data

Catch-ment	Parameter c_1					
	2/d	2/m	3/d	3/m	4/d	4/m
Jardine	1.08	0.97	1.03	1.02	0.80	2.65
Babinda	1.57	1.41	1.10	0.88	0.82	0.81
Cainable	2.87	2.66	1.97	2.71	2.67	2.21
Styx	1.12	2.80	1.98	1.57	1.13	1.82
Allyn	2.54	1.90	2.07	2.13	0.87	0.88
Corang	1.68	1.44	1.17	1.16	1.70	0.93
Tooma	0.80	0.81	0.82	0.83	0.80	0.83
Nariel	1.42	1.27	0.80	0.80	0.85	1.02
Dandong	2.82	2.96	2.43	2.28	2.40	2.57
Bass	1.31	1.33	1.33	1.29	1.36	1.37
Forth	1.09	1.14	0.96	0.92	0.98	0.86
Davey	0.86	0.83	0.81	0.80	0.87	0.80
Scott	2.84	2.44	2.15	1.88	1.70	1.64
Stones	2.74	3.00	2.66	2.95	2.01	2.01
Canning	2.45	2.99	2.97	3.00	2.79	1.77

It should be noted that a comparable result was obtained by Kokkonen and Jakeman (2001), who for demonstration purposes experimented by raising the value of PE_k in (12) to the fourth power. Using data from a small (0.49 km^2) humid zone (2220 mm annual rainfall) catchment in the US, they found that the fit of Model 2 to the runoff data was as good as when (12) was used without modification, but the seasonal variation of modelled actual evaporation E_k was substantially altered. They concluded that 'To avoid accepting a model which simulates evapotranspiration poorly, evapotranspiration data, or other constraining information, should be used wherever possible in the calibration process'.

5. CONCLUSIONS

Data from deep weighing lysimeters offer a means by which algorithms relating actual evaporation to PE and soil water can be compared. The small data set used in this study suggests that the conceptually superior algorithm (2) performs marginally better than (1) at site scale, and there is a suggestion of a similar result from the modelling at catchment scale.

The application of 4 models to 15 catchments indicate that equally good results, in terms of modelling daily stream flow , can be obtained by using average monthly PE data in place of daily data. Furthermore, in some catchments, particularly in lower rainfall areas, the best results are obtained when the PE data are multiplied by an unrealistically high factor.

It must be concluded that current models of the catchment drying process do not perform in the way they have been conceived, and a new approach is required.

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