The Third Wave: Mathematisation and Samuelson's *Economics*

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Abstract: It is said that there have been three great 'waves' in modern economics: the Keynesian revolution, the imperfect-competition revolution, and the mathematical and econometric revolution. Since 1945, the mathematical emphasis in economics has increased, with Paul A. Samuelson having a leading role in this revolution. Samuelson believes strongly in the importance and power of mathematics to aid in the progress of the economics discipline. His contribution to the mathematisation of economics is undisputed, and his use of mathematics in economics is widespread. Over the fifty years, 1948 to 1998, Samuelson has written sixteen editions of an introductory economics textbook, *Economics* (from the Twelfth Edition onwards with co-author William Nordhaus). An introductory economics textbook is more than a disseminator of information, but a physical object with specific content, presented in a particular way. This paper asks the question: has the increased formalism in the economics discipline been reflected in introductory economics textbooks, specifically the sixteen editions of Samuelson's *Economics*? Samuelson's introductory economics textbook, *Economics*, has spanned the last fifty years of modern economics. It does depict the increasing use of graphical analysis to present and teach the principles of economics. However, it does not reflect the trend towards an increasing use of numerical or algebraic analysis, in the economics discipline, and Samuelson's own preference for mathematical expression as a language in economic theory.

Keywords: Mathematisation; introductory economics; formalism

1. INTRODUCTION

It is said that there have been three great 'waves' in modern economics: the Keynesian revolution, the imperfect-competition revolution, and the mathematical and econometric revolution. Feiwel [1982a, p. 4] asserts that: "Samuelson made the greatest splash in the 'third wave' of modern economics...mathematization."

Since 1945, the mathematical emphasis in economics has increased, with Paul Samuelson having a leading role in this revolution. The change in methodology towards a more mathematical approach, appeared to give economics its real 'scientific' status. This was further advanced with the introduction in 1969 of a Nobel Prize in Economic Science.

This paper looks at Samuelson's place in the mathematisation revolution, and how this revolution is portrayed over fifty years (1948-1998) and sixteen editions of his introductory economics textbook *Economics*. Has the increased formalism in the economics discipline been reflected in introductory economics textbooks, specifically the sixteen editions of Samuelson's *Economics*?

2. SAMUELSON AND MATHEMATICS

"To those who accuse him of the mathematization of economics, Samuelson answers; 'that is one of the mortal sins for which I shall have to do some explaining when I arrive at heaven's pearly gates'" [Feiwel, 1982b, p. 169].

To Fischer [1987, p. 234] Paul Samuelson, more than any other economist, has "brought economics from its pre-1930s verbal and diagrammatic mode of analysis to the quantitative mathematical style and methods of reasoning that have dominated for the last three decades." In 1970, Samuelson won the Nobel Prize in Economic Science, the first American to do so. The citation for his Nobel Prize reads: "for scientific work through which he has developed static and dynamic economic theory and actively contributed to raising the level of analysis in economics science" [Lindbeck, 1985, p. 40]. Samuelson [1998, p. 1376] says of himself: "I was vaccinated early to understand that economics and physics could share the same formal mathematical theorems...while still not resting on the same empirical foundations and certainties." This connection to physics, and the scientific nature of economics is evident with the bulk of Samuelson's work collected under the title *The Collected Scientific Papers of Paul A Samuelson.*

As a graduate student at Harvard it was Samuelson who strove to clarify and progress economic theory using mathematical techniques. Samuelson's PhD thesis, *Foundations of Economic Analysis* published in 1947, used mathematics to present and elaborate on economic theory. Samuelson's approach in *Foundations* was a distinct change from the neo-classical approach of Marshall. Marshall put the role of mathematics in a secondary position, and warned the economics profession against putting literary propositions into mathematical form. It was Samuelson at the beginning of *Foundations* who regarded that "this dictum should be 'exactly reversed'; it is the effort of converting *essentially mathematical* propositions into *literary* form that is wasteful and 'involves...mental gymnastics of a peculiarly depraved type'" [Breit and Ransom, 1982, p. 112]. Thirty-five years later Samuelson published an enlarged and updated edition of *Foundations.* He saw that this "new stone caused no great ripples in the pond of modern mainstream economics [1998, p. 1378]. By 1983 we were all, so to speak, mathematical economists; and several hundred specialized books were available to cover each corner of up-to-date economics."

To Samuelson mathematical methods, when used correctly, enable the vision and analysis of ever more realistic and complicated hypotheses, rather than making theory more abstract. Briet and Ransom [1982, p. 112] observe that "Samuelson sees mathematics as much more than a mere device to clarify verbal arguments. 'With the assistance of mathematics,' he observes, 'I can see a property of the ninety-nine dimensional surfaces hidden from the naked eye.' Mathematics, in other words, can reveal aspects of economic theory which are not apparent from intuition alone." Samuelson seemed to have an aptitude to construct and portray economic concepts using mathematical models. It may be argued that one of Samuelson's major contributions to economics lies in the development of mathematical tools and applying them to economic theory. He views mathematics as a language, and a tool used to solve problems, not an end in itself. Using mathematical methodology Samuelson has been able to extend economic theory in a number of different areas, and to help solve some of the basic problems in economics.

Samuelson, the scientist, wanted to create a more rigorous, and important role and position, for economics in the field of science.[1] Through the incorporation of mathematics, and terminology borrowed from the physical sciences, he attempted to legitimise economics as a science, and create a unified body of mainstream economic thought. Pearce and Hoover [1995, p. 198] point out that:

> "Science, for Samuelson, is not just a matter of naïve realism; it also relies on a neutral and generally applicable analytical framework: 'The important thing is to provide the analytical machinery that will enable the reader to arrive at, and defend, his own opinion, and, what is hardly less important, to understand the position of those with whom he most disagrees' [Samuelson, 1948, p. vi]."

Samuelson was a leading figure in the 'third wave' in modern economics. Even as an undergraduate student he saw the possibilities of mathematics in improving and expanding the economics discipline. Throughout his academic career he has researched and published extensively utilising mathematical methodology – extending it into areas of economic theory previously not touched. Despite Samuelson's obvious love of the mathematical language and methodology, he never failed to make himself and his work understandable to economists, non-economists, and students.

3. MATHEMATICS IN *ECONOMICS*

"...who could have known that the young author of a number of profound and original papers in mathematical economic theory would not only continually create similarly fundamental analytical works but also produce a pedagogical masterpiece that was to become one of the most successful and influential economics texts of all time?" [Bergson, 1982, p. 335].

[1] "All men are mortal. Scientists are men. Ergo scientists are mortal. But science is immortal. Just as poetry is that which escapes in translation, scientific knowledge is that Cheshire residue which remains after you have boiled off the scientist cats" [Samuelson, 1975, p. 9].

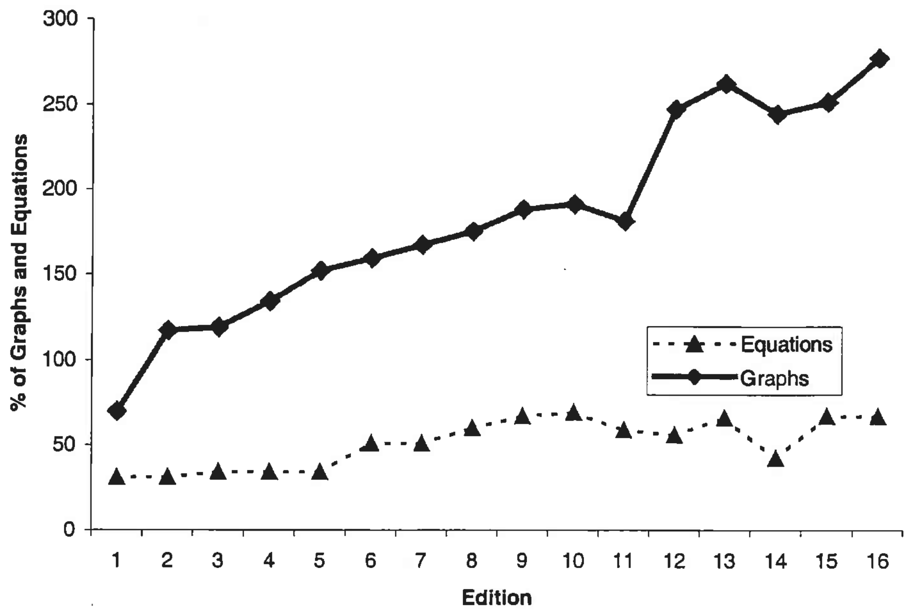

Figure 1. Graphs and equations as percentage of standardised pages

The use of graphs and algebra in introductory economics textbooks is today a 'fact of life;' graphs in particular are employed extensively. Mathematical techniques are used to present (and extend) economic ideas and to simplify (at times) wordy explanations. The questions are: How has Samuelson's predilection for a mathematical approach manifested itself in *Economics*? Has the amount of space given to graphs and equations increased over time?

Critics such as Bell [1988] say that graphs and equations take up too much space in introductory economics textbooks.[2] Others feel that Samuelson has increased the mathematical content in his textbook at the expense of written explanations. Sobel [1980, p. 106] observes that: "The book [*Economics*] remained quite readable, but, true to his natural bent, Samuelson attempted to wean students from words and towards numbers." By examining the first sixteen editions of *Economics,* the nature of the mathematical presentation Samuelson uses to demonstrate the principles of economics can be evaluated. This section first takes an overall look at the number of equations and graphs in the textbook, before examining the use of equations and graphs separately.

Hart [1948, p. 911] reviewed the First Edition of *Economics* and noted that: "The use of mathematics is held to an irreducible minimum: algebraic formulas are eliminated, diagrams are few and for the most part very clearly labelled; most of the mathematics is arithmetical illustration, with lavish verbal explanations." However, as later editions were published *Economics* was considered to have "a greater degree of intellectual sophistication" [Carvellas, et al, 1996, p. 227], in terms of its analytical rigor and theoretical presentation. Has *Economics* become more mathematical since the First Edition in 1948? Figure 1 presents the number of equations and graphs in *Economics*, and the number of equations and graphs as a percentage of standardised pages.[3]

[2] Bell [1988, p. 137] says: "Too much space goes to graphs and equations, with no facts at all, or to pedagogical examples with cooked-up numbers."

[3] To obtain the data the words from fifty pages from each edition were counted, then averaged to get the total average number of words per page. Every tenth page was counted except the first and last pages of a chapter, end-of-chapter question pages, the index, and glossary, to reach fifty pages. Standardised pages were calculated using the formula: standardised pages = (total words per page x total pages)/((16^{th} edition total words x total pages) x total pages of the edition).

From Figure 1, it can be seen that there are more than twice the number of equations in the Sixteenth Edition [1998] of *Economics* as compared to the First [1948], and nearly four times the number of graphs as compared to the First Edition. The average number of equations in an edition of *Economics* is 51.2,[4] which is 20.2 more than the First Edition, and 15.8 less than the Sixteenth. The average number of graphs is 183.4,[5] which is 113.4 more than the First Edition, and 93.6 less than the Sixteenth Edition. Taken on face value it can be said that there has been an increase in the mathematical content of Samuelson's introductory economics textbook, and in the presentation of the material. Samuelson, it seems, has presented the economic principles with a greater emphasis on equations and graphs.

Isolating the total number of equations, and then examining them as a percentage of standardised pages, will provide a better view of the algebraic (numerical) approach in *Economics*. There are thirty-one equations in the First Edition [1948] of the textbook; this increases through to the Tenth Edition [1976] with sixty-nine equations. There is a decline in the number of equations in the textbook from the Tenth Edition to the Twelfth Edition [1985], before it increases again in the Thirteenth [1989]. The Fourteenth Edition [1992] has twenty-four fewer equations than the previous edition, a marked decline. The number of equations, however, increases in the Fifteenth Edition [1995] to sixty-seven, and remains at this number in the Sixteenth Edition [1998].

By examining the number of equations as a percentage of standardised pages, however, it is possible to see another view. In the First Edition [1948], 9.6 percent of the pages feature an equation. This percentage declines through to the Fifth Edition [1961]. The percentage of standardised pages with an equation then rises in the Fifth to Tenth Editions [1961-1976], before decreasing again in the Eleventh and Twelfth Editions [1980-1985]. The Thirteenth through to Sixteenth Editions [1989-1998] have an increasing percentage of equations per edition; this is despite the dramatic fall in the total number of equations in the Fourteenth Edition.[6] The Sixteenth Edition has an equation on 8.6 percent of its pages, which is a one percent decline from the First Edition. So how accurate is it to say that there has been an increasing emphasis on the algebraic presentation of economic ideas and concepts in *Economics*? In fact, there has been very little change in the amount of algebraic content, especially when considering that the First Edition is the edition with the greatest number of equations as a percentage of its standardised pages.

Repeating this exercise and isolating the total number of graphs, it is possible to get a better perspective of the graphical approach in *Economics*. The number of graphs in *Economics* has had a more dramatic rise than the number of equations over the course of the sixteen editions. The First Edition [1948] has seventy graphs scattered through it. This count steadily increases to the Thirteenth (and largest) Edition [1989] where 262 graphs are used. The number of graphs then drops back in the Fourteenth Edition [1992].[7] There is, however, an increase in the number of graphs in the Sixteenth Edition [1998]. This is also the edition that has the highest number of graphs with 277. Overall, there has been an increase of 207 graphs from the First to Sixteenth Edition, almost four times the number of graphs from the First Edition.

When looking at the number of graphs as a percentage of standardised pages there is a more stable relationship over time. In the First Edition [1948] twenty-two percent of its pages contain a graph. This percentage stays around the region of twenty to twenty-four percent until the Eleventh Edition (1980), where it dips to 16.7 percent. The number of graphs as a percentage of standardised pages, however, increases in the Fourteenth to Sixteenth Editions [1992-1998].

The Sixteenth Edition includes a graph on 35.5 percent of its pages, an increase of 61.4 percent from the First Edition. A further indication of the greater use and importance of graphical analysis is also evident by the addition of an appendix in the Twelfth Edition [1985], "How to Read Graphs." This appendix has remained through to the Sixteenth Edition of *Economics*.

[4] The median number of equations in an edition of *Economics* is 53.5.

[5] The median number of graphs in an edition of *Economics* is 178.

[6] This coincided with the removal of nearly a quarter of the pages of the textbook from the Thirteenth Edition.

[7] This coincided with the removal of nearly a quarter of the pages of the textbook from the Thirteenth Edition.

It is clear that as topics have changed over the editions, so has the nature of their presentation. Some topics have a greater propensity for mathematical expression than others. This means that depending on the emphasis and changing nature of the principles of economics, there may be greater (or less) mathematical expression of concepts. An example is the rudiments of Marxian economic thought that appears in the Ninth through Eleventh Editions [1973-1980] of *Economics*, and is presented using algebraic notation.[8] The trend in Samuelson's *Economics* is an increasing emphasis on graphical presentation and analysis, and a more limited role for algebraic manipulation. Graphs in particular are used to extend the written theory and aid in its understanding.

Another point to consider is the marketability of the textbook to economics departments and students. If it is too mathematical then there may be a hesitation (or disinclination) to adopt the textbook in an introductory economics course, with many students coming to university less mathematically prepared and able than in the past [Bartlett, 1995]. Therefore, textbooks that are too mathematically rigorous may not be the preferred textbook for an introductory economics course, especially with the drive to increase student numbers in economics courses.

What does Samuelson say about the presentation of introductory economics to students? Does he believe the mathematical style of presentation needs to be utilised at lower levels of economics teaching? Samuelson explains in the First and Sixth Editions:

- "This [PPF] can be illustrated quantitatively by simple arithmetic examples and by means of geometrical diagrams. It has been found that diagrams and graphs are important visual aids in many parts of economics, so that a little care at the beginning in understanding them will be rewarded manyfold later on" [Samuelson, 1948, p. 17].
- "Although every introductory textbook must contain geometrical diagrams, knowledge of *mathematics* itself is needed only for the higher reaches of economic theory. Logical reasoning is the key to success in the mastery of basic economic principles, and shrewd weighing of empirical evidence is the key to success in mastery of economic applications" [Samuelson, 1964, p. 6].

Samuelson's own philosophy of downplaying the algebraic presentation of economic concepts at the introductory level explains his approach in *Economics*. Graphs, if presented clearly, go a long way in aiding students' understanding of economic concepts, for example, costs and revenues in the theory of the firm. Samuelson caters for students' need to make sense of the mathematical tool that is extensively used with the provision of an appendix on how to read and understand graphs.

Overall, it is not accurate to say that *Economics* has become more mathematically rigorous. The number of equations as a percentage of standardised pages is actually higher in the First Edition [1948] than the Sixteenth Edition [1998]. There has been very little change in the amount of algebraic content over the sixteen editions of *Economics*, but graphs have become an increasingly important tools used to present economic theory. The introduction of an appendix, "How to Read Graphs," aids the reader in their understanding of graphs and their use. Possibly part of the drive to keep textbooks 'thin' is in no small way a factor in the increasing use of graphs to conserve space and reduce lengthy verbal discussions and explanations.

4. CONCLUSION

Mathematics is seen as "a young man's game...And it is historically true that the really great achievements in pure mathematics have usually been made by men when they were in the first brilliance of their youth" [Samuelson, 1954, p. 380].

Since the 1930s people such as Tinbergen, Frisch, Klein, Kalecki, Hicks, Allen, Lange, and Samuelson have represented a large part of the core of the 'third wave' of modern economics – mathematisation. The economics discipline has come to have a more scientific nature and perception, and Nobel Prize winning laureates in Economic Science have existed since 1969.

Paul Samuelson believes strongly in the importance and power of mathematics to aid in the progress of the economics discipline. His contribution to the mathematisation of economics is undisputed, and his use of mathematics in economics is widespread. This first American economist to win a Nobel Prize in Economic Science, however, has always retained

[8] This section appears in *Economics* in the Ninth through to Eleventh Editions [1973-1980] as an appendix, "Rudiments of Marxian Economics."

the ability to communicate his research to his colleagues and the general public alike. Hurwicz [1970, 720] highlights this when saying: "By explaining, in nontechnical terms, the relationship of [Samuelson's] often quite technical contribution to the work of others, he educates a broader public and encourages further development."

Samuelson's First to Sixteenth Editions of his introductory economics textbook, *Economics*, have spanned the last fifty years of modern economics. It does depict the increasing use of graphical analysis to present and teach the principles of economics. However, it does not reflect the trend towards an increasing use of numerical or algebraic analysis, in the economics discipline, and Samuelson's own preference for mathematical expression as a language in economic theory. Samuelson's [Samuelson, 1998, p. xxvii] philosophy is unambiguous when he writes: "...*after* you have mastered instruction in so-called microeconomics and macroeconomics, there will remain no mysteries. If it doesn't make good sense, it isn't good economics."

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