Anthropogenically-Induced Fractal Tidal Waves in Coastal Aquifers at Bundaberg, Queensland, Australia

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Abstract: In this paper, we show that salt concentration profiles are periodic fractals which are induced anthropogenically in coastal aquifers of Bundaberg, Queensland, Australia. The fractal dimensions are computed and mapped using the Hentschel and Procaccia'a generalised fractal dimension model and borehole conductivity data collected at Bundaberg. We also show that the fractality of the salt profiles and associated tidal waves are predominant in urban and industrial areas where frequent human activities are present which impose severe impact on the waveforms, and that the fractal tidal waves are heterogeneous as indicted by variable dimensions at different orders. Spectral analysis of the salt profiles and waves is also performed, and it is shown that multifractal analysis yields more consistent and reliable results than spectral analysis.

Keywords: **Fractal; Aquifer; Seawater**

1. INTRODUCTION

The topic of waves is a classic theme in hydrodynamics in general, and theories on tidal waves, in particular, are issues which are of great theoretical and practical importance [Bear, et al., 1999; Elmore and Heald, 1969; Lamb, 1945; Stoker, 1992]. In traditional theories waves are characterised by such simple terms as frequency, phase, and amplitude, and have not been examined as to whether they are fractal or not. In this paper we show that tidal waves as characterised by salt concentration profiles in a coastal aquifer are fractal as a result of anthropogenic effects.

In this paper we make use of the multifractal analysis, which is a more powerful tool to characterise the spatial heterogeneity of both theoretical and experimental fractal patterns [Grassberger and Procaccia, 1983], because the methodology does not neglect the fact that tidal waves are highly heterogeneous. Multifractal analysis was initially proposed for studying turbulence. In recent years multifractal analysis has been successfully applied in other fields including time series analysis [Canessa, 2000; Pastor-Satorras, 1997], modelling transport in porous media [see Wheatcraft and Tyler, 1988 for a brief review], financial modelling [Anh et al., 2000], and DNA, sequencing [Berthelsen et al., 1994; Yu and Anh, 2001].

In this paper, the fractal phenomenon in aquifers induced by tides in coastal regions is investigated using Hentschel and Procaccia's [1983] generalised fractal model and borehole data on salt water conductivity in the region. The fractal dimensions of periodic fractal waves in boreholes are computed and mapped to reveal the spatial variability of waveforms in the coastal region of interest.

The theoretical and practical implications of the findings include that fractal waveforms can be induced by human activities in different regions which can be compared, and spatial variability of fractal tidal waveforms in coastal regions can be mapped to reveal the areas of influence. The data used in the analysis was acquired by periodic logging of conductivity at given depths of different boreholes. The data shows clear periodicity at various locations in Bundaberg indicating periodic tidal influence on the coastal aquifers. In this region, human activities impose significant impacts on shallow aquifers, not only in terms of water being pumped out for various purposes, but in the way tides change their waveforms due to these activities. Intense human activities imposed on the region are concentrated in the urban areas and around some sugar mills where machinery appearance is frequent. In the rest of the region, ordinary farming is the major activity on the ground.

The analysis in the following section will show these impacts. In order to analyse the tidal and wave characteristics,. data acquired and hosted at the Queensland Department of Natural Resources is used. The data set used covers a coastal region in Bundaberg, Queensland. The borehole implemented for monitoring purposes is shown in Figure 1.

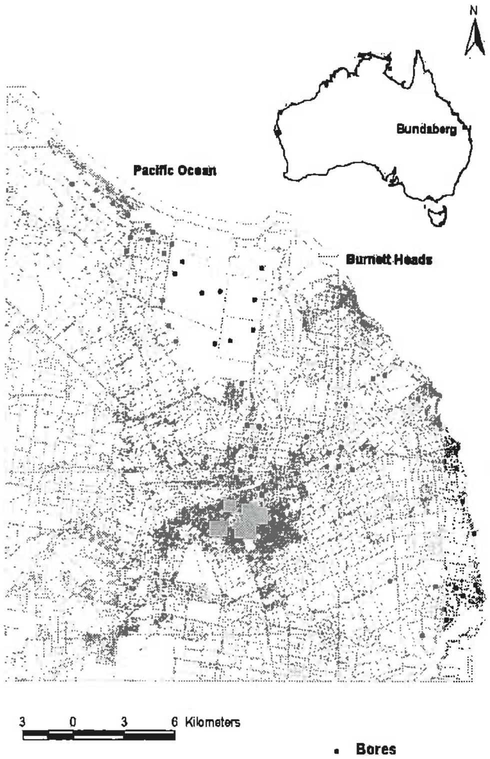

Figure 1. Location map.

2. FREQUENCY AND MULTIFRACTAL ANALYSES

Extensive data analysis shows that the periodicity of tides at different locations differs in wave amplitude, phase shift, and frequency. In order to reveal further the spatial variability of the tidal waves in terms of wave characteristics, both spectral analysis and fractal analysis of the tidal waves were carried out.

2.1. Frequency Analysis

If we view the tidal wave as a time series T_t, $t = 1,2,\ldots,N$, we can formulate the following for analysis.

First we define:

$$F_t = \frac{T_t}{\sum_{t=1}^{N} T_t} \qquad (1)$$

to be the frequency of T_t. It follows that $\sum_t F_t = 1$.

We now consider the discrete Fourier transform of the time series F_t, $t = 1,\ldots,N$, defined by:

$$F(f) = N^{-\frac{1}{2}} \sum_{t=0}^{N-1} F_{t+1} e^{-2\pi i f t} \qquad (2)$$

then

$$S(f) = |F(f)|^2 \qquad (3)$$

is the *power spectrum of* F_t.

It has been found in related studies [Robinson, 1974] that many natural phenomena lead to the power spectrum of the form $1/f^{\beta}$. This kind of dependence was named *1/f* noise, in contrast to white noise $S(f) = const$, i.e. $\beta = 0$.

Let the frequency f take k values $f_k = k/N$, $k = 1,\ldots,N$, then from the $\ln(S(f))$ *vs.* $\ln(f)$ graph, we can infer the value of β from the $\ln(S(f))$*vs.* $\ln(f)$ relationship above the low-frequency range. For example, the log power spectrum of salt concentration at boreholes B13500125 is shown in Figure 2, and β is derived to be 0.2065033.

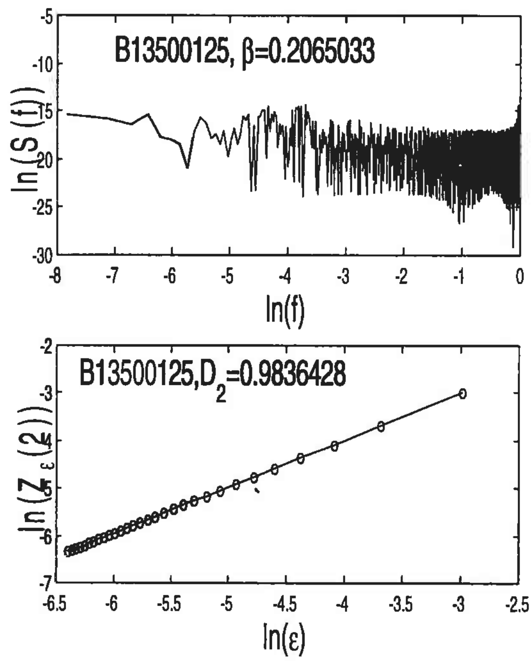

Figure 2. Spectral analysis of electric conductivity For borehole B13500125, and linear fit of D_2.

The figures shows that spectral analysis yields less consistent results which are similar to the findings reported for coastal lines [Southgate and Moller, 2000], we do not intend to discuss spectral results in further detail.

2.2. Multifractal Approach

First, we examine the variability of salt concentrations measured at different times in the same borehole.

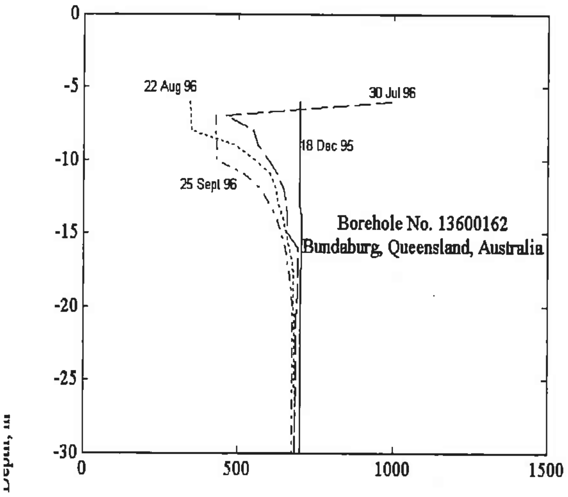

Figure 3. Variability of aquifer electric conductivity at different times

As shown in Figure 3, the variability of conductivity in the same borehole within about 20 metres below the ground surface is apparent. The shapes of the salt profiles are irregular for various times at various depths. This type of irregularity at different boreholes in the region cannot be simply compared using any conventional methods. However, with the computed fractal dimensions of the salt concentration profiles at different boreholes, one can compare the differences.

Now we define a measure μ on [0,1) by $d\mu(x) = Y(x)dx$, where

$$Y(x) = N * F_t$$

when

$$x \in \left[\frac{t-1}{N}, \frac{t}{N}\right) \qquad (4)$$

It is easy to show that $\int_0^1 d\mu(x) = 1$, and

$$\mu\left(\left[\frac{t-1}{N}, \frac{t}{N}\right)\right) = F_t .$$

The most commonly used numerical procedure for implementing multifractal analysis is the fixed-size box-counting algorithms [Halsy et al., 1986]. In the one-dimensional case, for a given measure μ with support $E \subset \boldsymbol{R}$, we consider the partition sum

$$Z_\varepsilon(q) = \sum_{\mu(B)\neq 0} [\mu(B)]^q , \qquad q \in R \qquad (5)$$

where the sum runs over all different nonempty boxes B of a given side ε in a grid covering of the support E, that is,

$$B = [k\varepsilon, (k+1)\varepsilon) \qquad (6)$$

We further define exponent $\tau(q)$ as

$$\tau(q) = \lim_{\varepsilon \to 0} \frac{\log Z_\varepsilon(q)}{\log \varepsilon} \qquad (7)$$

and the generalised fractal dimensions of the measure are defined as

$$D_q = \frac{\tau(q)}{(q-1)} \qquad \text{for } q \neq 1 \qquad (8)$$

and

$$D_q = \lim_{\varepsilon \to 0} \frac{Z_{1,\varepsilon}}{\log \varepsilon} \qquad \text{for } q = 1 \qquad (9)$$

Where

$$Z_{1,\varepsilon} = \sum_{\mu(B)\neq 0} \mu(B) \log(B) \qquad (10)$$

D_0 is called the fractal dimension. With our normalisation, D_0 is equal to 1.0 for all time series. D_1 is called the information dimension, and D_2 the correlation dimension.

D_q values subject to positive q are indications of relevance to the regions where the measure is big, i.e., data with a high probability. D_q values subject to negative q deal with the structure and the properties of the most rarefied regions of the measure.

In this paper, information dimension, and correlation dimension, i.e., $q = 1, 2$, are given only. With the aid of Eqs. (8) and (9), the generalised fractal dimensions of the salt concentration profiles measured at various boreholes in various locations are computed, and the statistics of the data are summarised in Table 1.

Table 1. Statistical summary of the D_1 and D_2

D_1	D_2
Minimum: 0.617977	Minimum: 0.472164
Maximum: 0.999184	**Maximum: 0.997680**
Mean: 0.790727	Mean: 0.754243

The simple statistical data only shows the range of the variability, but does not offer much information about the spatial variability itself. In order to visualise the spatial variability of these two dimensions, namely D_1 and D_2, the following graphs have been generated by plotting the results using an appropriate coordinate. The data set used in the mapping is for the boreholes shown as dots in Figure 1.

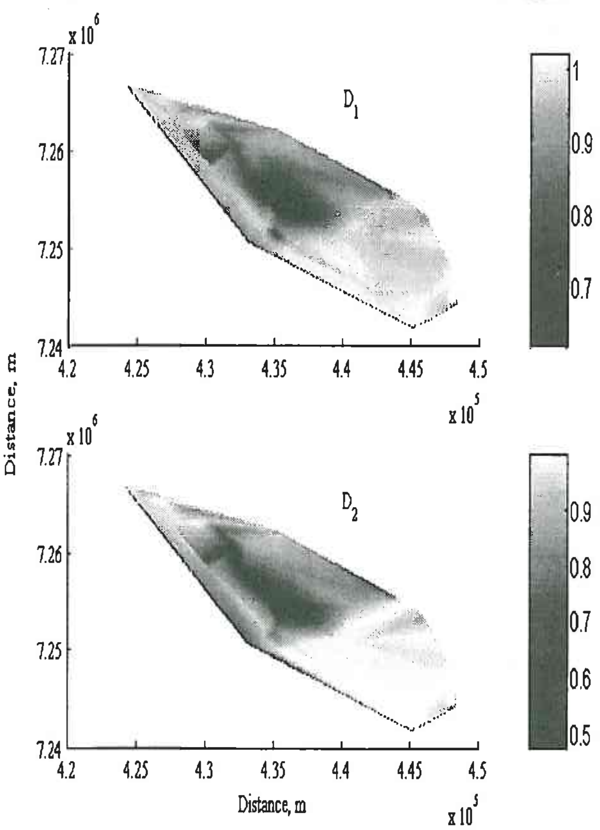

Figure 4. D_1 and D_2 dimensions for Bundaberg, Queensland, Australia.

D_1 and D_2 show very similar patterns of fractal dimensions in the region. The regions with nearly uniform values of D_1 or D_2 are ordinary farmland. The region with the lowest value of D_1 or D_2 is in the centre of the urban area, and the surrounding lower values of D_1 or D_2 correspond to either intensively populated areas or sugar mills. In the estuary where river flows interact with tides, fractality increases which implies more complexity.

3. CONCLUSIONS

In the preceding presentation, we have shown that human activities reshaped tidal waveforms in the coastal aquifer resulting in fractal phenomenon in tidal waveforms in the aquifers. The reshaping of the tidal waves is revealed using Hentschel and Procaccia's [1983] generalised fractal model and borehole electric conductivity data. Spatial variability is also investigated by mapping the fractal dimensions in the region.

In the analysis, D_1 and D_2 are used only which represent different entities, but they are both consistent throughout the region as shown.

Compared to spectral analysis, multifractal analysis offers a more consistent and reliable tool to investigate various fractal phenomena in nature.

The methodology and examples presented in the paper show that multifractal analysis is a useful tool for investigating external forces which reshape waveforms. The data used in this paper is for tidal waves, but the methods can be directly applied to other types of waves.

The method and results presented in the analysis can be used for various purposes. Immediate applications include (i) the classification of the areas subject to different external disturbances characterised by the differences in fractal dimensions; (ii) partitioning the regions in order to assign different values of parameters to a model for quantitative analysis, and (iii) providing useful reference for planning purposes etc.

It should be noted that the areas subject to random disturbances from river flows near estuaries and river banks also tend to have fractal phenomena. However, we are particularly interested in anthropogenic effect in this paper.

4. ACKNOWLEDGMENTS

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