Modelling Stream Recession Flows

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Abstract: It is well known that a semilog plot of flow against time during a hydrograph recession results in a curve which is concave upwards. This has been explained previously by the assumption of a power relationship between groundwater storage and its outflow to the stream, with no recharge occurring during the period after cessation of surface runoff. The current work is based on the alternative hypothesis of a linear groundwater system with a continuing inflow from the vadose zone. This leads to the development of stream recession equations with time-varying inputs of various forms, including those derived from lysimeter data. These and the 'no recharge' models are compared using data from 22 Australian benchmark catchments. The recession equation of the IHACRES model, which takes the form of the sum of two exponential functions, is also examined in this context, and is found to provide a very good fit to the data. From the performance of the models which include recharge, it is concluded that significant recharge does continue through recession periods, and should be accounted for in conceptual models of the rainfall-runoff process. In the later stages of a recession, the groundwater system may have significant losses due to evaporation from the stream surface, transpiration from phreatophytes, or leakage to underlying strata, resulting in a semilog curve which is convex upwards. Such losses can be incorporated in the recession equation, and the magnitude of the losses can be quantified.

Keywords: Recession; Base flow; Groundwater recharge; Hydrographs; Rainfall-runoff models

1. INTRODUCTION

After surface flow has ceased, the recession part of a streamflow hydrograph is regarded as resulting from groundwater discharging into the stream. The equation most used for this period is

$$Q = Q_o e^{-t/\tau} = Q_o k^t \qquad (1)$$

where Q_o, Q_t are the flows at times 0 and t, τ is the turnover time of the groundwater storage, and k is the recession constant for the selected time units. The first form has a long history [Boussinesq, 1877; Horton, 1933; Maillet, 1905], while the second was popularised by Barnes [1939].

Equation (1) results from a linear storage, in which the groundwater storage S is related to the stream flow Q by

$$Q = S / \tau = a\,S \qquad (2)$$

where $a = 1/\tau$.

While this equation would be expected from an aquifer in which there is little variation in flow depth, in unconfined flow situations a two-dimensional hydraulic analysis [Chapman, 1963; Werner and Sundquist, 1951] suggests a non-linear relationship [Coutagne, 1948] of the form

$$Q = a\,S^n \qquad (3)$$

where n would be expected to lie between 1 and 2.

This results [Chapman, 1999] in a recession equation of the form

$$Q = Q_o\,[1 + (n-1)t/\tau_o]^{-n/(n-1)} \qquad (4)$$

where $\tau_o = S_o/Q_o$ is now the turnover time at time 0.

Wittenberg [1994] fitted this equation to 21 streams in Germany and China, and obtained values of n ranging from 1.1 to 9.1, but stated that a value of 2.5 was 'typical'. Chapman [1999] obtained mean values of n from 1.6 to 3.2 for 11 benchmark catchments in Eastern Australia, and suggested that the high values might be attributed to horizontal convergence of the groundwater flow paths.

Both these approaches are based on the assumption that no significant groundwater recharge occurs during a recession period, that is, all recharge occurs during periods of surface runoff. This assumption is enshrined in many popular rainfall-runoff models, such as MODHYDROLOG [Chiew et al., 1993] and AWBN [Boughton, 1993].

It is the main purpose of this paper to question whether this assumption is valid, as consideration of soil physics would suggest that the duration of recharge would be considerably longer than that of surface runoff. Wu et al. [1996] emphasised the critical importance of water-table depth in determining the lag between rainfall and groundwater recharge. With shallow water-tables, recharge events correspond closely with individual rainfall events. As the depth to groundwater increases, correspondence tends to be with groups of rainfall events, and trends towards a single annual process. With a very deep water-table, variations in water-table depth become imperceptible.

Even at a depth of only 1.5 m, deep drainage has been estimated as occurring continuously over 4-6 weeks under wheat and lupin crops in a deep sandy soil at Moora, WA [Anderson et al., 1998].

Similar conclusions can be drawn from considering percolation from the base of deep lysimeters. Figure 1 shows a typical percolation hydrograph for a lysimeter 2.4 m deep at Coshocton, Ohio. It will be noted that the peaks in percolation correspond to very high rainfalls or groups of rainfall events, and that the percolation continues at a rate of about 1 mm/d for periods of over 50 days.

It is therefore apparent that streamflow recession equations should take account of recharge continuing through some or all of the recession period, and such equations are developed in the next section. These conceptual equations, and those based on the 'no recharge' assumption, will be compared with the equation derived from the systems approach in the linear module of the IHACRES model [Jakeman and Hornberger, 1993], which can be expressed as

$$Q = Q_o[f_q e^{-t/\tau_q} + (1 - f_q) e^{-t/\tau_s}] \quad (5)$$

where τ_q, τ_s are the time constants for quick and slow flow respectively, and f_q is the fraction of quick flow in the stream flow at time 0.

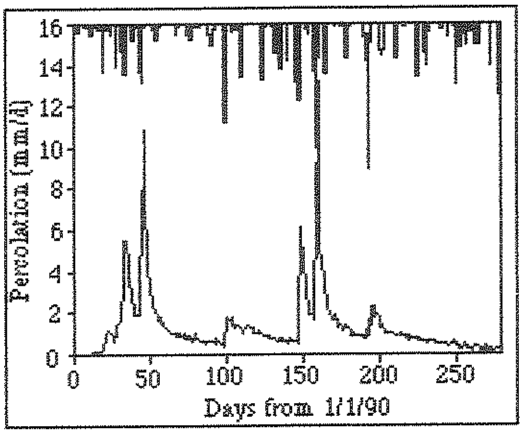

Figure 1. Percolation and rainfall at Lysimeter Y101D at Coshocton, Ohio. Rainfall scale is 10 times percolation scale.

2. RECESSIONS WITH RECHARGE

Assuming the groundwater behaves as a storage of volume S with time-varying input of recharge R and output of streamflow Q, the water balance equation is

$$\frac{dS}{dt} = R - Q \quad (6)$$

If the storage is linear, combining (6) and (2) gives

$$\tau \frac{dQ}{dt} + Q = R \quad (7)$$

for which the general solution is

$$Q = Q_o e^{-t/\tau} + \frac{e^{-t/\tau}}{\tau} \int_0^t R e^{t/\tau} dt \quad (8)$$

This solution will now be evaluated in terms of 3 different assumptions about the time variation of R.

Model 1: It is assumed that variation in R is sufficiently small that it can be replaced by its mean value $\bar{R}$. The solution of (8) then is

$$Q = (Q_o - \bar{R}) e^{-t/\tau} + \bar{R} \quad (9)$$

Model 2: It is assumed that R has an exponential decline, as in the generalised SFB model [Ye et al., 1997]. The solution is

$$Q = Q_o e^{-t/\tau} + \frac{R_o \tau^*}{\tau^* - \tau} (e^{-t/\tau^*} - e^{-t/\tau}) \quad (10)$$

where R_o is the recharge at time 0, and τ^* is the time constant for the declining recharge.

Model 3: In tests of a range of algorithms for prediction of percolation from lysimeters [Chapman and Malone, 2001], it was found that good results were obtained from a modification of the drainage algorithm in the water balance form of the IHACRES model [Evans and Jakeman, 1998], expressed as

$$R = b\, e^{-\,CMD/a} \quad (11)$$

where CMD is the catchment moisture deficit, and a and b are constants. The variation in time of R, when there is no infiltration into the soil store, is

$$R = \frac{a R_o}{a + R_o t} \quad (12)$$

Combining (12) with (8) results in

$$Q = Q_o e^{-t/\tau} + \frac{e^{-t/\tau}}{\tau}\, a\, e^{-c} [W^*(c + t/\tau) - W^*(c)] \quad (13)$$

where $c = a/R_o\tau$ and the function W* differs only by a constant from the well function W used in groundwater pumping tests, and is defined by

$$W^*(u) = \ln u + u + \frac{u^2}{2.2!} + \frac{u^3}{3.3!} \cdots\cdots$$

Model 4. A further model can be developed for the situation where evaporation losses become significant, and the stream flow decreases to zero. If the evaporation loss in such a period is taken as constant, the model is readily derived from (9) as

$$Q = (Q_o + E)\, e^{-t/\tau} - E \quad (14)$$

3. DATA AND CALCULATIONS

The data used in this study were the stream flow records in the data set of Australian catchments prepared by Chiew and McMahon [1993]. The locations of the gauging stations are shown in Figure 2, and details of the catchments are given in Table 1. Flows for the 24h period up to midnight were used for the Queensland catchments, and up to 9 am for the other stations. Daily flows in ML were converted to an equivalent depth in mm over each catchment. Recession periods were identified as sections of the hydrograph, of at least 10 days' duration, that were close to linear on a plot of log Q against time. This minimum duration was selected in view of the number of parameters in the models ranging up to 4. No recessions of this duration (in most cases of any duration) were found for 6 catchments (Ref. Nos. 5, 11, 17, 20, 23 and 27). Each recession for the other catchments was fitted to each of the models defined in Sections 1 and 2, using as an objective function the sum of squares of differences between the logs of the observed and modelled flows. This objective function gives equal weight to a given proportional error in the modelled flows, which corresponds to a roughly proportional error in the measurement of stream flows. The value of Q_o was taken as a parameter to be optimised. The optimisation technique was a modification of the simplex technique [Nelder and Mead, 1965].

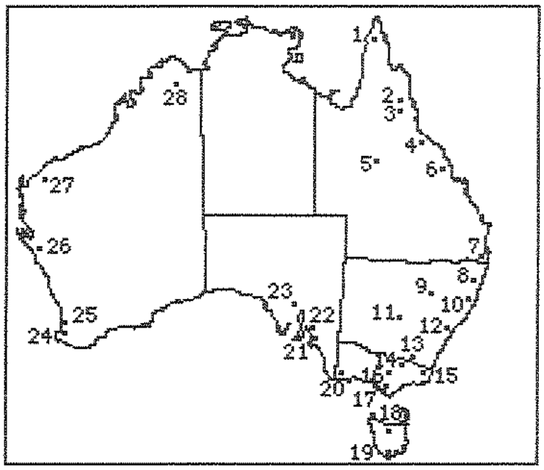

Figure 2. Location of catchments listed in Table 1, from Chiew and McMahon [1993].

4. RESULTS

The models have been compared in two ways. Table 2 gives the number of events in which each model gave the best fit to the data. In Table 3, a score based on ranks has been used, with a score of 5 for the best fitting model and a zero score for the worst. Both tables show that each model can on occasions provide the best fit to the data, but in general the models based on the 'no recharge' assumption (Equations 1 and 4) perform less well than those which assume a continuing recharge (Equations 9, 10 and 13). There does not seem to be any pattern in these results in relation to catchment area or annual rainfall.

Table 1. Details of catchments used in study, from Chiew and McMahon [1993].

Map ref.	National No.	Catchment name	Area (km^2)	Mean rain (mm)	Record length (y)
1	927001	Jardine R. at Telegraph Line	2500	1700	16
2	111105	Babinda Ck at The Boulders	39	5400	16
3	113004	Cochable Ck at Powerline	93	2400	13
4	118106	Alligator Ck at Allendale	69	1100	15
5	915001	Mitchell Grass at Richmond	3	450	13
6	120204	Broken R. at Crediton	41	2100	15
7	145103	Cainable Ck at Good Dam Site	41	900	13
8	206001	Styx R. at Jeogla	163	1300	8
9	420003	Belar Ck at Warkton	133	1100	12
10	210022	Allyn R. at Halton	215	1200	8
11	412093	Naradhan Ck at Naradhan	44	450	11
12	215004	Corang R. at Hockeys	166	800	10
13	401554	Tooma R. above Tooma Reservoir	114	1700	9
14	401212	Nariel Ck at Upper Nariel	252	1200	11
15	222213	Suggan Buggan R. at Suggan Buggan	357	800	14
16	403218	Dandongadale R. at Mating North	182	1300	11
17	227219	Bass R. at Loch	52	1100	12
18	315006	Forth R. U/S Lemonthyme	311	2000	12
19	317001	Davey R. D/S Crossing River	686	2100	17
20	238208	Jimmy Ck at Jimmy Creek	23	650	20
21	503502	Scott Ck at Scotts Bottom	27	950	16
22	505517	North Para R. at Penrice	118	550	12
23	509503	Kanyaka Ck at Old Kanyaka	180	300	12
24	612005	Stones Brook at Mast View	15	1000	11
25	616065	Canning R. at Glen Eagle	544	800	11
26	701003	Nokanena Brook at Woottachooka	229	400	10
27	708009	Kanjenjie Ck Tributary at Fish Pool	41	400	13
28	809312	Fletcher Ck at Frog Hollow	30	650	11

Table 2. Number of events in which each model provided the best fit to the data.

Catch-	No.	Equation No.					
ment	events	1	4	5	9	10	13
1	5	1	0	4	0	0	0
2	43	0	2	11	14	14	2
3	24	1	4	6	10	1	2
4	13	0	0	5	4	3	1
6	15	0	2	6	5	1	1
7	7	0	1	4	1	1	0
8	26	0	1	5	11	3	6
9	16	0	0	4	9	2	1
10	14	0	1	2	4	3	4
12	26	0	0	11	7	4	4
13	15	0	0	5	5	2	3
14	35	2	1	11	11	0	10
15	18	0	0	7	4	5	2
16	20	0	0	6	7	4	3
18	18	0	1	8	8	1	0
19	13	0	0	8	5	0	0
21	13	0	0	5	6	1	1
22	4	0	0	3	1	0	0
24	13	0	0	11	1	1	0
25	16	0	0	8	4	3	1
26	7	0	0	5	2	0	0
28	4	0	0	1	2	1	0
All	365	4	13	136	121	50	41

Table 3. Scores based on ranking of fits of the models to the data.

Catch-	No.	Equation No.					
ment	events	1	4	5	9	10	13
1	5	8	16	20	14	4	14
2	43	10	69	162	143	167	101
3	24	10	57	81	89	41	84
4	13	4	20	50	51	32	41
6	15	5	37	48	54	36	49
7	7	8	19	29	19	15	16
8	26	7	52	84	104	47	96
9	16	10	23	51	67	47	42
10	14	10	26	44	45	41	45
12	26	12	39	107	98	54	81
13	15	10	27	51	58	27	53
14	35	38	53	120	128	67	122
15	18	20	18	64	63	51	56
16	20	5	34	74	76	47	64
18	18	3	27	75	73	40	55
19	13	0	32	56	50	26	32
21	13	0	26	52	53	29	37
22	4	1	6	17	12	14	13
24	13	6	9	61	34	50	35
25	16	10	21	63	54	59	33
26	7	5	14	31	26	22	8
28	4	1	9	15	15	8	12
All	365	183	634	1355	1326	924	1089

The calibrated value of $\bar{R}$ in Model 1 ranges from a minimum of 0.01 mm/d for catchments 25 and 26 to a maximum of 3.1 mm/d for catchment 2, with an overall average of 0.4 mm/d. Expressed as a proportion of the stream flow at time 0 (Q_0), the values range from 0.11 to 0.35, with an average of 0.25. There is some association between success of the recharge models and higher values of R.

The results also show that the IHACRES model, which takes the form of the sum of two exponential functions, provides as good a fit to the data as the best of the conceptual models. In Table 4, the values of the time constants derived from these recession periods are compared with values obtained by Ye [1996] from calibration of the continuous streamflow hydrograph over the whole period of record for the Queensland catchments, and 2 years for the other catchments. The results show the great variability from event to event of time constants derived from recessions.

For the catchments in Eastern Australia, the values of τ_q derived by Ye are significantly lower than those obtained from the recessions, but values for the remaining catchments are generally within one standard deviation of the mean for the recessions, as are most of the values for τ_s.

5. THE EVAPORATION LOSS MODEL (MODEL 4)

The only catchment in which the loss effect was evident over a duration suitable for model fitting was the Canning River (Ref. 25) in the period from October of each year. Figure 3 shows that Model 4 fits the data in this period very well, even when there is evidence of some minor 'freshes' in the stream flow. The average value of E for 12 such periods is 0.0025 mm/d, which is 23% of the average flow at the start of the period. Taking the potential evaporation at this time of year to be 5 mm/d, the effective area evaporating at this rate is 0.0025 / 5 of the catchment area of 544 km^2, which is 27 ha. This is a plausible estimate of the area of river bed contributing to evaporation loss.

6. DISCUSSION

The observation that semilog plots of hydrograph recessions are generally concave upwards is reinforced by the low scores of (1), the straight line solution. While the nonlinear groundwater storage puts curvature into the model, the shape of the curve does not match the data as well as the sum of two exponential recessions (5) or the models which assume continuing recharge (9,10,13).

Table 4. Comparison of quick and slow time constants obtained by Ye [1996] from calibration of the IHACRES model, with mean and SD of values obtained from fitting to recessions. * indicates only a quick flow component was identifiable in the IHACRES model.

Catch-	No.	τ_q			τ_s		
ment	events	Ye	Mean	SD	Ye	Mean	SD
1	25	4.8	35.2	22.8	68	230	141
2	43	0.7	6.8	5.7	30	58	45
6	15	0.2	9.7	10.7	37	65	54
7	7	0.2	7.0	4.7	62	39	18
8	26	1.5	11.4	7.9	26	75	61
9	16	0.6	7.1	4.9	9	34	22
10	14	1.2	11.4	12.5	64	56	50
12	26	1.7	5.8	3.0	*	38	18
14	35	5.0	14.0	8.1	76	99	50
16	20	4.5	6.1	3.5	89	51	40
18	18	2.2	4.3	2.1	*	34	28
19	13	2.3	2.3	1.3	*	18	16
21	13	0.9	5.0	3.8	57	45	31
22	4	2.2	3.4	1.6	*	16	3
24	13	4.5	3.0	1.8	*	13	4
25	16	7.4	4.0	1.5	*	19	18
26	7	1.6	3.5	2.6	*	17	10

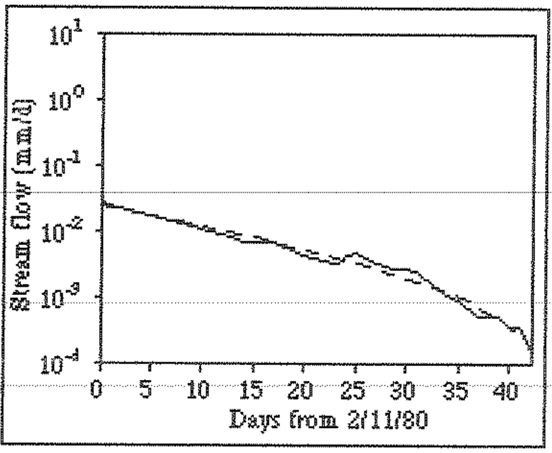

Figure 3. Fit of Model 4 (dashed line) to end of year stream flow in Canning River.

The differences between the quick flow time constants determined from the recessions and those obtained by calibration of the IHACRES model should be expected, as the selection of the recession periods has deliberately removed those parts of the hydrograph in which surface flow was apparently occurring. To model the whole of a recession, from the point of inflection onwards, would require an additional term for the surface flow. The concept of a recession being characterised by the sum of three exponential functions (representing surface runoff, interflow, and base flow) dates back to Barnes [1939]. Although this results in a continuous curve, engineering hydrologists [Klaassen and Pilgrim, 1975] have attempted to fit three straight lines to the data in order to determine the relevant time constants. No attempt appears to have been made to verify this procedure by comparing the resulting continuous curve with the data.

Although Models 2 and 3, which assume a declining rate of recharge, performed well, the assumption of a constant recharge rate used in Model 1 scored slightly higher. It was noted that the calibrated values of τ^* in Model 2 and 'a' in Model 3 were both high, indicating a very slow rate of decline.

It shuld be noted that an equation of similar form to Model 4 [Chapman, 1999] was derived for a leaky catchment, where part of the groundwater has its outflow outside the catchment boundary. It is apparent that the two forms of loss cannot be distinguished by hydrograph analysis.

7. CONCLUSIONS

This study supports the conclusion that groundwater recharge continues, at a constant or slowly declining rate, through periods of base flow. This suggests that conceptual models of the rainfall-runoff process should provide for rapid accessions to groundwater during periods when the soil store is saturated, followed by a continuing recharge until the next event.

Where transmission losses due to evaporation, or leakage from the catchment groundwater, may occur, Model 4 provides an algorithm for inclusion of this factor in a conceptual model.

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