Mathematical Modelling for Optimal Operation of Integrated Multireservoir Systems: New Approach

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Abstract: In this work a mathematical model for optimization of the operation of cascade system of 5 dams on Karoon River together with one dam on Dez River taking into account the stochastic nature of the flows is considered. The basin is multipurpose on the usage side expected to meet agricultural, industrial, drinking, environmental demands. The system has the potential to generate some 10,000GWH per year at the present. Part of this hydroelectric power is firm energy used in the power network and part of it is used to stabilize the network in peak consumption. An integrated stochastic dynamic programming model is defined for the system with the objective of maximizing the hydropower and meeting the usage demands. The physical data of land topography and reservoirs structural data together with water demands are built in constraints into continuity equation throughout the system hence reducing the number of variables, further reduction in calculations achieved via discretizing the data of integrated system, cutting down the value sets of the state vectors, this has also caused the fast convergence of the procedures in the model, sweeping all cases in a cycle of 5 years. The advantage of this model over the existing models available is that in the optimal solution each strategy is given with the risk of non-realization therefore providing the operation managers with some useful information in decision-making. Also given are the full detailed distributions of solution sets, i.e. distribution of storage heights inflow values, and amounts of releases. The model is tested by simulation of stream flows for precision and robustness. A software package including stream flow forecast is developed and tested for use.

Keywords: Modelling; Optimization; Multireservoir; Stochastic Dynamic Programming; DSS

1. INTRODUCTION

Planning the operation of a river basin with even a single major reservoir and several downstream powerhouses is a complex problem. Planning the operation of two or more river basins with several major reservoirs and attached powerhouses can be a very complex problem because: (a) Future inflows are uncertain (b) The optimal releases from the reservoirs in the cascade depend not only on their own storage and upstream reservoir release, but also on the local inflows to the storages (c) Cross correlation among concurrent stream flows are often high, autocorrelations vary in magnitude, and neither the cross nor the autocorrelations should be neglected (d) Stream flow forecasts are often used by the operating policy (e) because of daily and monthly large variation on the power demands, the optimization of power production is not always straight forward. Various optimization and simulation models were proposed and many authors reviewed several applications of these models. Karamooz and Houck [1982] develop reservoir-operating rules by deterministic optimization. Karamooz and Houck [1987] compare the deterministic model with stochastic model. Karamooz and Houck [1992] used an implicit stochastic optimization method for a two-reservoir system.

Wurbs et al. [1985] presented a bibliography for various optimization and simulation models and listed several applications of reservoir operation problems. Wurbs [1993] reviewed the reservoirs systems simulation and optimization models.

Simonovic [1992] presented a short review of mathematical models used in reservoir management and operation. Yang and Read [1999] develop constructive DP, which is successfully applied to optimize releases in a two-reservoirs in New Zealand. Takyi and Lence [1999] developed a surface water quality management model. Lund and Guzman [1999] reviewed the single purpose operation policies in parallel and in series for water supply, flood control, hydropower and water quality. Chandramouli and Raman [2001] give a model for multireservoir operation with dynamic programming and neural networks.

Many of the above mathematical models use stream flow forecast and as they seldom have access to a perfect stream flow forecast, they employ a deterministic model adapted to the problem [Yeh, 1985]. An alternative to this approach is to use explicit stochastic dynamic programming [Loucks et al., 1981; Yakowitz, 1982; Bras et al., 1983; Steadinger, 1984]. This approach generates operation strategies for every possible reservoir storage state in each period. Unfortunately, this representation of the system is often simplified to make the algorithm work [e.g. Saad and Turuen, 1988], and recent papers purpose interpolation scheme to reduce computation [Foufoula-Georgio et al., 1988].

In this work, an explicit stochastic dynamic programming model is developed for multireservoir system in parallel and in series without any simplifications. First the hydraulic model for the inflows of the integrated system is developed. Then the distribution structure of the hydraulic system is constructed and an explicit backward stochastic dynamic programming model is developed. The Chapman-Kolmogrov equations for the stationary state solutions are set up. A decision support system based on optimum strategies given [Badamchizadeh et al., 2001].

As a case study, this model is used to develop operation rules for 'Dez' and 'Karoon' dams considered separately as single reservoirs [Hashemiparast et al., 1999]. It is also applied to a two-reservoir system of 'Dez' and 'Karoon' dams [Hashemiparast et al., 2000]. In this work the results of application of this model for optimization of operation rules for the integrated system of six dams -'Karoon IV', 'Karoon III', 'Karoon I', 'Godar Landar', 'Gotvand' and 'Dez' dams- are reported.

Decision support systems (DSS) for different scenarios of Dez-Karoon project i.e. one-reservoir, two-reservoir, ..., six-reservoir are developed from the optimum strategies tables. These decision support systems are user-friendly software that for each reservoir; given time, storage level and forecast inflow, provides the optimum release both in amount and flow units plus the risk of nonrealization of that strategy [Badamchizadeh et al., 2001].

2. STOCHASTIC DYNAMIC PROGRAMMING MODEL

2.1. Introduction

First, to make a hydraulic model of the system the stream flows are given discrete codes. The system constraints developed using the given data for various water demands and physical specifications of the whole basins including reservoirs. Then a system of the mass balance equations for the upstream releases and local inflows are setup using these codes of inflows and the same codes for the reservoir levels.

Using the historical data and cross correlation relations amongst the local inflows and releases the conditional distribution for the inflows into various storages are estimated. Then the complete backward stochastic dynamic programming model is set up for the whole system, the minimization of the expected value of the sum of squares of deviations from the demands plus sum of squares of deviations from the flood control levels used as objective function for the system. In solving the SDP model set up above with an insight to maximization of hydropower, we have applied a selection strategy in stages of SDP. Thus at each stage the solution sets is contracted as we go along. Then the model is run to get the stationary state solution and the Chapman-Kolmogrov equations are setup and solved to get probabilities of the stationary state solutions.

2.2. System Considered

The system considered for this case study is 'Dez' and 'Karoon' basins project. This is a multipurpose, multireservoir project with interbasin transfer of water from one basin to another. The system is situated in southwest of Iran and 'Khozestan Water & Electricity Organization' manages operation of the system. This system interlinks three main basins namely 'Dez', 'Karoon', and 'Karkheh'. Figure 1 shows the location plan of the system in this study. Table 1 gives the names of the reservoirs and the respective numbers referred to them in the system. Table 2 gives the main stream and local flows and their cross correlation coefficients in the system. The analysis of hydraulic data yielded the stationary seasonal inflow structures satisfying the Ergodic property given in Table 3. The coded inflow matrix with the given seasons is tabulated for 'Karoon I' inflows in Table 4. The coded storage levels for given seasons using the same scale of 400^{MCM} presented in Table 5.

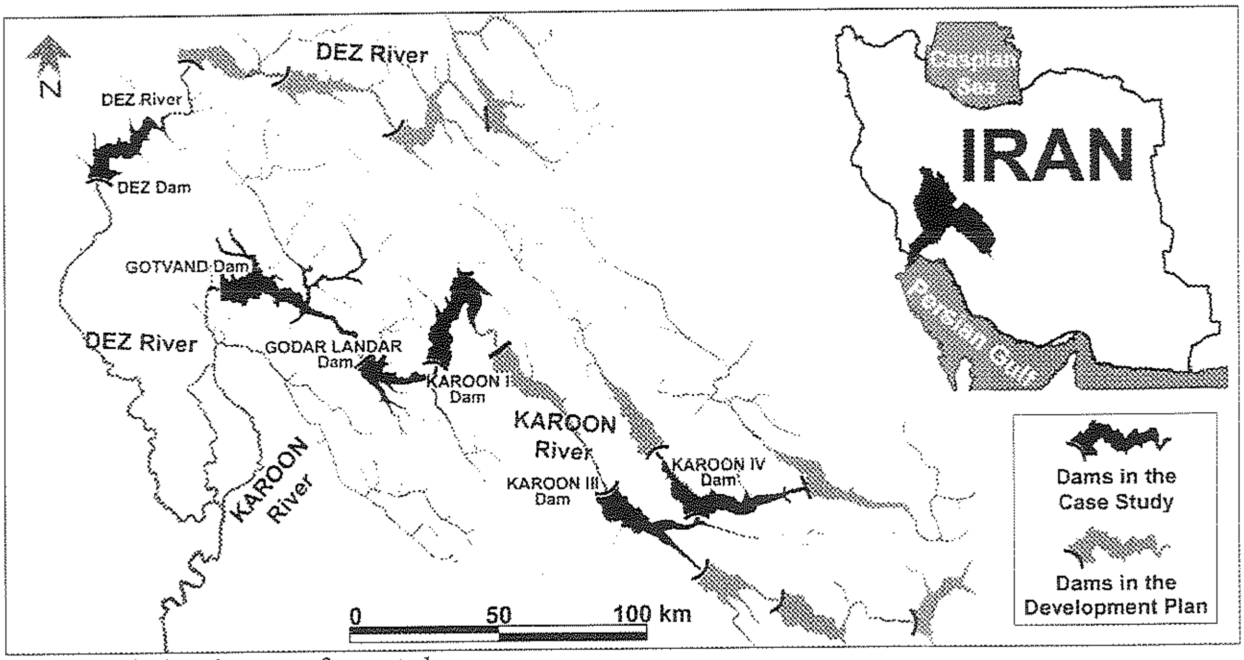

Figure 1. The location map of case study.

Table 1. Reservoirs in 'Dez' and 'Karoon' project.

Reservoir Name	Referred as in tne Model
Karoon IV	1
Karoon III	2
Karoon I	3
Godar Landar	4
Gotvand	5
Dez	6

Table 2. Cross correlation between coefficients main streams and local inflows.

Local Inflow	Main Stream	Coeff
Morghab & Shoorandika Rivers	Karoon I	0.99
Morghab & Shoorandika Rivers	Dez	0.94
Karoon I	Dez	0.94
Karoon I – Karoon III local inflows	Karoon III	0.99
Morghab & Shoorandika Rivers	Karoon I – Karoon III local inflows	0.99
Morghab & Shoorandika Rivers	Dez	0.94
Khersan River	Karoon IV	0.88
Karoon I – Karoon III local inflows	Karoon IV	0.98
Karoon I – Karoon III local inflows	Dez	0.94
Morghab & Shoorandika Rivers	Karoon III	0.99

Table 3. Water year seasons.

Saeson 1	20 Feb – 21 May
Saeson 2	22 May – 21 Jul
Saeson 3	22 Jul – 22 Oct
Saeson 4	23 Oct – 19 Feb

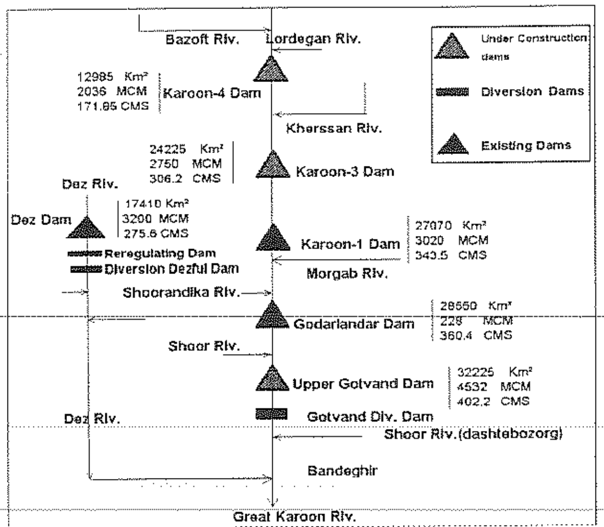

Figure 2. The system plan of 'Dez' and 'Karoon' project.

Table 4. Inflow matrix (400 MCM).

Season 1	Season 2	Season 3	Season 4
32.4	29.6	29.7	84.9
57.2	40.8	45.1	130.7
82	52	60.5	176.5
106.8	63.2	75.9	222.3
131.6	74.4	91.3	168.1

Table 5. Storage level matrix.

Season 1	Season 2	Season 3	Season 4
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8

2.3 Model Formulation

The recursive backward stochastic dynamic programming equations for six-reservoir problem for a given time period 't' follow the notations definition below. Let:

S_t^k	The levels of kth reservoir in the system (k=1,2,…,6), vector variable; t=N represents the normal storage level
Q_t^k	The inflows to kth reservoir or the kth midbzsin inflow in the system (k=1,2,…,6), vector variable;
R_t^k	The Release from kth reservoir in the system (k=1,2,…,6), vector variable;
D_t^k	Total down stream demands projected to the kth reservoir, vector variable;
$B^{(K)}(t\ ,S_t,R_t)$	Benefit of the kth reservoir with given storage level S and release R, vector variable;
$B^{K,m}(t,S_t,R_t)$	Benefit of the kth & mth reservoirs with given storage levels S^k & S^m and release R, vector variable;
$f_n^{(t)}$	Accumulative benefit up to nth stage in cascade, rectangle array variable;
$f_n^{(t)}(k,m)$	Accumulative benefit up to nth stage in parallel of the kth & mth reservoirs , rectangle array variable;

The set of mass balance equations for the system are:

$$S_t^1 = S_{t-1}^1 + Q_t^1 - R_t^1$$

$$S_t^2 = S_{t-1}^2 + Q_t^2 - R_t^2; \quad Q_t^2 = R_t^1 + Q_t^{1\_2}$$

$$S_t^3 = S_{t-1}^3 + Q_t^3 - R_t^3; \quad Q_t^3 = R_t^2 + Q_t^{2\_3}$$

$$S_t^4 = S_{t-1}^4 + Q_t^4 - R_t^4; \quad Q_t^4 = R_t^3 + Q_t^{3\_4}$$

$$S_t^5 = S_{t-1}^5 + Q_t^5 - R_t^5; \quad Q_t^5 = R_t^4 + Q_t^{4\_5}$$

$$S_t^6 = S_{t-1}^6 + Q_t^6 - R_t^6; \quad Q_t^6 = R_t^5 + Q_t^{5\_6}$$

$$R_t^k \geq 0 \qquad ; k = 1,2,3,4,5,6$$

For each reservoir in series connection with the other reservoirs the 'benefit' for each stage is

$$B^{(K)}(t\ ,S_t,R_t) = (S_t^K - S_N^K)^2 + (R_t^{K)} - D_t^{(K)})^2$$

In addition, the recursive equations are

$$f_1^{(t)} = B_t^{(k)}(S_t^k, R_t^k)$$

$$f_n^{(t)} = \underset{R_t^k, S_t^k}{Min}(B^{(k)}(t), S_t^k, R_t^k) + E[f_{n-1}^t]$$

With the stationary state condition

$$\left| f_n^{(t)} - f_{n-1}^{(t)} \right| < \varepsilon\ .$$

and for any two reservoirs in parallel connection we have

$$B^{K,m}(t,S_t,R_t) = (S_t^K - S_N^K)^2 + (S_t^m - S_N^m)^2 + (R_t^K + R_t^m - D_t)^2$$

$$f_1^{(t)}(k,m) = B_t^{(k,m)}(t\ ,S_t^k, R_t^k)\ \forall k,m$$

$$f_n^{(t)}(k,m) = \underset{t\ R_t^k, S_t^k}{Min}(B_t^{(k,m)}(t\ ,S_t^k, R_t^k) + E[f_{n-1}^t(k,m)]\forall k,m$$

With the stationary state condition

$$\left| f_n^{(t)}(k,m) - f_{n-1}^{(t)}(k,m) \right| < \varepsilon\ .$$

The stationary state probabilities are the solutions of following set of Chapman-Kolmogrov equations:

$$\pi_{t+1} = \pi_t . P_t$$

$$\sum_i \pi_t^{(i)} = 1$$

where

$$P(S_t^k = s^k) = P_t$$

in addition, $\pi_t^{(i)}$ is the ith element of π_t vector of probabilities in the stationary state, used for calculating risk of optimum strategies.

The set of Chapman-Kolmogrov equations for stationary state solution for six-reservoir problem with five storage levels and five inflows for each season contain 4204 equations with 4200 unknowns. As this matrix is not 'full rank' after 'block decomposition', the generalized inverse method is used to solve these equations. Column six in table six gives these probabilities corresponding to each strategy. In column seven the risk of nonrealization of each strategy calculated using these stationary state probabilities.

3. RESULTS

The model generates operation policy or release decision for every reservoir storage level and for every inflow forecast value for the nodes in the system for given season including the risk of not realization of each strategy. The expected values of the optimum release tables are used to generate optimum rule curves for each reservoir. These rule curves were used to calculate the average annual hydropower production, then in comparison with the historical data of power production at the existing hydropower plants at 'Dez' and 'Karoon I' dams these figures showed an increase of 8%-11%. Table 7 presents comparison of hydropower production under two policies for 'Karoon I' power plant. Later using the optimum strategy tables the short term daily release policies for operating dams calculated, and the daily release policy decisions for the past three years were compared with the present policy operated data. This showed again improved performance of operation using optimum strategies; both in term of covering demands and power production. Decision support systems for different scenarios of the system i.e. 1-reservoir, 2- reservoir, ... , 6-reservoir are developed from the optimum strategies tables. These decision support systems are user-friendly software that given time, storage level and forecast inflow, provide the optimum release both in amount and flow units. Table 6 presents the optimal decision rules for operation of 'Karoon I' dam and Figure 3 shows the present policy vs. the optimum policy in terms of storage maneuver, release, demand, and inflow. For brevity, the similar results for five other reservoirs are not reported here. An ARIMA time series model developed for the forecast of inflows and this together with DSS programs is made user-friendly software, a powerfull tool for operation of integrated multipurpose multireservoir systems, a simple demo is given at the following address: (http//:www.wrpm.i8.com).

4. CONCLUSION

The stochastic dynamic programming model (SDP) for operation management or planning study of complex hydrosystems provides probably the most compatible mathematical model for water resources problems. Today with the help of fast computers proper hydraulic analysis of river systems and running complex and sophisticated recursive routines required by SDP is feasible. Again using up to date skills of computer programming one no longer needs so-called shortcuts and approximations to cut down computation time. The computer programs developed for 6-reservoir model takes less than 60 minutes to run on a dual processor Pentium III 550 MHZ PC. This model for the planning purposes gives much more accurate results than conventional simulation models. Since today, we want to have an optimum design at planning stage rather than feasible one. On the operation management side, one achieves nearly the ultimate optimized decision rules with using accurate model of the system, this version of multipurpose multireservoir SDP model have the ability to incorporate thermal and gas power plants in order to optimize the integrated power network of a district.

Table 6. Optimum strategy tables of 'Karoon I' dam in the 6-reservoir system.

	S1	I	S2	R	PR	Risk
Season 1	1	1	4	5.49	.3846	0
	1	2	5	9.07	.1538	.4359
	1	3	5	13.65	.1026	.7436
	1	4	5	18.23	.0256	.8718
	1	5	5	22.81	.0513	.9487
	2	2	5	10.07	.1026	.5897
	2	4	5	19.23	.0256	.8974
	3	1	5	6.49	.0256	.3846
	3	2	5	11.07	.0513	.6923
	3	3	5	15.65	.0256	.8462
	3	4	5	20.23	.0256	.9231
	5	1	5	8.49	.0256	.4103
Season 2	4	1	3	3.97	.362	0
	4	2	3	5.51	.0226	.4359
	5	1	4	3.97	.0739	0
	5	2	4	5.51	.2851	.4359
	5	3	4	7.05	.1538	.7436
	5	4	4	8.59	.0513	.8974
	5	5	5	9.13	.0513	.9487
Season 3	3	1	1	4.96	.3	0
	3	2	1	6.08	.0345	.3846
	3	3	1	7.2	.0501	.6154
	4	1	1	5.96	.0846	.3
	4	2	1	7.08	.1963	.4191
	4	3	1	8.2	.1806	.6655
	4	4	1	9.32	.1026	.8462
	5	5	2	10.44	.0513	.9487
Season 4	1	1	1	3.24	.6415	0
	1	2	1	5.72	.2051	.4615
	1	3	2	7.2	.1282	.7179
	1	4	3	8.68	.1282	.8462
	1	5	5	9.16	.0256	.9744
	2	2	1	6.72	.0513	.6667

Legend of Table 6

S1	Index of Present Storage Level
S2	Index of Next Storage Level
I	Index of Inflow Matrix
R	Release * 400^{MCM}
PR	Probability of Realization

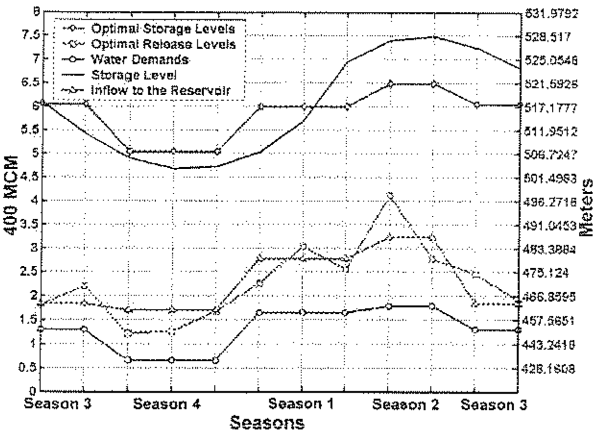

Figure 3. Present operating policy vs. optimum operating policy.

Table 7. Comparison of hydropower production.

TIME	OHP (MWH)
Season 1	1269.7
Season 2	917.3
Season 3	1047.4
Season 4	777
Total Energy under this Model	4011.3
Mean Hydropower Production Past 5 years	3598.3 MWH
Percentage Optimized	%11.47

5. ACKNOWLEDGEMENT

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